

# Reduced Entries Algebraic Magic and Semi-Magic Squares of Orders 4, 6, 8 and 10

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*The whole work as pdf files is available at author's sites:*

<https://numbers-magic.com/?p=16145>

*This work is without use of any kind of programming language*

## Abstract

*This work brings **magic** and **semi-magic** squares of orders 4, 6, 8 and 10 for **reduced entries**. By **reduced** or **less entries**, we understand that instead of normal  $n^2$  entries of a magic square order  $n$ , we are using less numbers. Moreover, in these situations the entries are no more sequential numbers. These **entries** are **non-sequential positive** and **negative numbers**. In some cases, these may be **decimal** or **fractional** values depending on the orders of a magic squares. In this work we bring different ways of writing **magic** and **semi-magic** squares of orders 4, 6, 8 and 10. These are based on three types of magic squares, i.e., **cornered**, **single-digit bordered** and **double-digit bordered** magic squares. The pandiagonal magic squares for reduced entries for the orders 4, 6, 8 and 10 are also considered. In case of order 6, we have given two different ways of **pandiagonal** magic squares. Most of the results are with **magic rectangles**. In each case, these are considered with **equal width** and **length**. Previously, the author brought similar kind of work [30, 31, 32, 33, 34, 35] for the orders 3 to 12 specially for the **dates** and **days** of the year 2025, where the **dates** are few **entries** and **days** are the **sums** of magic squares. This work is revised and enlarged version of previous work.*

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# 1 Introduction

This work brings **magic** and **semi-magic** squares of orders 4, 6, 8 and 10 for **reduced entries**. By **reduced** or **less entries**, we understand that instead of normal  $n^2$  entries of a magic square order  $n$ , we are using less numbers. Moreover, in these situations the entries are no more sequential numbers. These **entries** are **non-sequential positive** and **negative numbers**. In some cases, these may be **decimal** or **fractional** values depending on the orders of a magic squares. In this work we bring different ways of writing **magic** and **semi-magic** squares of orders 4, 6, 8 and 10. These are based on three types of magic squares, i.e., **cornered**, **single-digit bordered** and **double-digit bordered** magic squares. The pandiagonal magic squares for reduced entries for the orders 4, 6, 8 and 10 are also considered. In case of order 6, we have given two different ways of **pandiagonal** magic squares. Most of the results are with **magic rectangles**. In each case, these are considered with **equal width** and **length**. The table below give the number of reduced entries algebraic magic squares constructed in this work except the order 4. It is due to F. Gaspalou [3].

Orders	Magic Squares	Pandiagonal Magic Squares	Semi-Magic Squares	Total
4	1	1	0	2
6	2	2	1	5
8	5	1	3	9
10	9	1	10	20

The author [30, 31, 32, 33, 34, 35] also worked on similar kind of work but from different point of view. This work is for the magic squares of orders 3 to 12 for the **dates** and **days** of the year 2025, where the **dates** are few entries and **days** are the sums of the magic squares. This work is revised and enlarged version of previous work.

## 2 Magic Square of Order 4

**Example 2.1.** *The famous Khajuraho magic square of order 4 is given by*

		34	34	34	34
	7	12	1	14	34
34	2	13	8	11	34
34	16	3	10	5	34
34	9	6	15	4	34
	34	34	34	34	34

In this case, we shall present two different forms of less entries magic squares of order 4. One is **pandiagonal** and another is normal magic square of order 4.

## 2.1 Reduced Entries Magic Squares of order 4

We shall present two different forms of **reduced entries** magic squares of order 4. One is **pandiagonal** and another is normal magic square of order 4.

**Result 2.1.** A *pandiagonal* magic square of order 4 with *reduced entries* is given by

A1	A2	A3	S-A1-A2-A3
A4	S-A1-A2-A4	A1-A3+A4	A2+A3-A4
S/2-A3	A1+A2+A3-S/2	S/2-A1	S/2-A2
S/2-A1+A3-A4	S/2-A2-A3+A4	S/2-A4	A1+A2+A4-S/2

### • Details

The magic square given in Result 2.1 is **pandiagonal**. We observe that, in the last two lines of this result the sum  $S$  is divided by 2. It means that in case of **magic sum** as even number, the **entries are integers**, and in case of **magic sum** as odd number, the **entries are decimal numbers**. The magic square of order 4 is due to F. Gasparou [3]. See the examples below

**Example 2.2.** Let's consider following two examples of reduced entries magic squares of order 4 based on the Result 2.1



	pan	335	335	335	335			pan	224	224	224	224
335	11	22	40	262	335		224	21	22	44	137	224
335	33	269	4	29	335		224	5	176	-18	61	224
335	127.5	-94.5	156.5	145.5	335		224	68	-25	91	90	224
	163.5	138.5	134.5	-101.5	335			130	51	107	-64	224
4x4	335	335	335	335	335		4x4	224	224	224	224	224

Both the examples are **pandiagonal**. The first examples is with decimal entries as it's sum is an **odd** number. The second example is with normal integers because it has a sum as **even** number.

The following result is a normal magic square of order with reduced entries. In this case, the entries are always integers independent of magic sum.

**Result 2.2.** Let's consider following magic square of order 4

A1	M+A4-A6-A1	A6-A3-A4	A3
A6	A4	A5	M-A4-A5-A6
A2+A3-A6	A1+A2-A5	M-A1-A2-A4	A4+A5+A6-A2-A3
M-A1-A2-A3	A5-A2-2*A4+A6	A1+A2+A3+2*A4-A5-A6	A2

## • Details

The magic square given in Result 2.2 is not a **pandiagonal**. It is a normal magic square. In this case the entries are always integers independent of magic sum even or odd numbers. The magic square of order 4 is also due to F. Gaspalou [3]. See the examples below.

**Example 2.3.** Let's consider following two examples of **reduced entries** magic squares of order 4 based on the Result 2.2

	mgc				70			mgc				101
	11	51	-11	19	70			11	82	-11	19	101
	31	23	27	-11	70			31	23	27	20	101
	3	-1	21	47	70			3	-1	52	47	101
	25	-3	33	15	70			56	-3	33	15	101
4x4	70	70	70	70	70		4x4	101	101	101	101	101

Both the examples give magic square of order 4. One is with even number magic sum and another with odd number magic sum.

There are much more way to write **reduced entries** magic squares of order 4. See below

**Result 2.3.** Let's consider following magic square of order 4 with different kind of *reduced entries*

A1	A2	S-A1-A6-A2	A6
A3	A4	A7	S-A4-A7-A3
A5+A6-A3	A1+A5-A7	S-A1-A5-A4	A4+A7+A3-A5-A6
S-A1-A5-A6	S-A1-A5-A4+A7-A2	2*A1+A5+A6+A4-A7+A2-S	A5

**Result 2.4.** Let's consider following magic square of order 4 with different kind of *reduced entries*

A1	A4	A6	S-A1-A4-A6
A2	S-A1-A2-A4	2*A1+A2+A3-A5+A6-S	S-A1-A2-A3+A4+A5-A6
A3	A5	S-A1-A3-A6	A1-A5+A6
S-A1-A2-A3	A1+A2-A5	S-A1-A2+A5-A6	A1+A2+A3+A6-S

**Result 2.5.** Let's consider following magic square of order 4 with different kind of *reduced entries*

A1	A3-S+A6	2*S-2*A3-A6-A1	A3
2*S+A4-A3-A6-A1	A4	A5	A3-A5-2*A4-S+A6+A1
A2+2*A3-2*S+A1-A4+A6	A1+A2-A5	S-A1-A2-A4	2*A4+A5+2*S-2*A3-A6-A1-A2
S-A1-A2-A3	A5-A2+2*S-A3-A6-A4-A1	2*A1+A2+2*A3-A5-2*S+A6+A4	A2

In the above three results the letter S represents the magic square of order 4. The readers can try to get some examples based on above three results.

### 3 Magic Squares of Order 6

**Example 3.1.** Let's consider following two magic squares of order 6

6	mgc							111
		17	22	11	24	29	8	111
		12	23	18	21	27	10	111
		26	13	20	15	35	2	111
		19	16	25	14	9	28	111
		4	3	36	30	6	32	111
		33	34	1	7	5	31	111
©IJT		111	111	111	111	111	111	111

6	mgc							111
		6	30	3	36	4	32	111
		35	17	22	11	24	2	111
		29	12	23	18	21	8	111
		27	26	13	20	15	10	111
		9	19	16	25	14	28	111
		5	7	34	1	33	31	111
©IJT		111	111	111	111	111	111	111

The first magic square is **cornered** and another the second one is **single-digit bordered**.

#### 3.1 Reduced Entries Magic Squares of Order 6

Based on **cornered** magic square below is a reduced entries of magic square of order 6.

**Result 3.1.** Let's consider a following magic squares of order 6 with reduced entries:

A1	M+A4-A6-A1	A6-A3-A4	A3	S-M-A10	A10
A6	A4	A5	M-A4-A5-A6	S-M-A11	A11
A2+A3-A6	A1+A2-A5	M-A1-A2-A4	A4+A5+A6-A2-A3	M-S+A10+A11+A12	2*(S-M)-A10-A11-A12
M-A1-A2-A3	A5-A2-2*A4+A6	A1+A2+A3+2*A4-A5-A6	A2	S-M-A12	A12
2*S-3*M+A1-A8+3*A4-A11+A10	S-M-A8	3*M-2*S-A1+2*A8-3*A4+A11-A10+A9	S-M-A9	(S-M)/2	(5*M-3*S)/2
2*M-S-A1+A8-3*A4+A11-A10	A8	3*S-4*M+A1-2*A8+3*A4-A11+A10-A9	A9	(5*M-3*S)/2	(S-M)/2

*Details:*

It is a **cornered** magic square of order 6 having magic square of order 4 at upper left corner. The two magic rectangles of orders  $2 \times 4$  are of equal width and length. The letters M and S represents the magic squares of orders 4 and 6 respectively. In order to avoid decimal entries, the magic sums M and S should be of same type, i.e., either **even** or **odd** numbers.

Below are two examples based on the Result 3.1.

**Example 3.2.** Let's consider following two examples:

1a	mgc							140
	21	87	-21	23	0	30		140
	26	24	25	35	63	-33		140
	19	18	43	30	-1	31		140
	44	-19	63	22	-2	32		140
	2	-74	131	1	15	65		140
	28	104	-101	29	65	15		140
	140	140	140	140	140	140		140

1b	mgc							151
	21	88	-21	23	10	30		151
	26	24	25	36	53	-13		151
	19	18	44	30	9	31		151
	45	-19	63	22	8	32		151
	12	-53	110	11	20	51		151
	28	93	-70	29	51	20		151
	151	151	151	151	151	151		151

The magic sums of above two examples are as follows:

**First Example**

$$M_{4 \times 4} = 110$$

$$S_{6 \times 6} = 140$$

**Second Example**

$$M_{4 \times 4} = 111$$

$$S_{6 \times 6} = 151$$

Above two examples are considered with magic sums as **even** and **odd** numbers. The magic rectangles sums are:

**First Example**

$$MR_{2 \times 4} = 30 \times 60$$

**Second Example**

$$MR_{2 \times 4} = 40 \times 80.$$

**Result 3.2.** Let's consider a following magic squares of order 6 with reduced entries:

$A_{19}+A_{20}+A_{21}+A_{22}+A_{23}-A_4-A_7-A_{11}-A_{15}$	A4	A7	A11	A15	$S-A_{19}-A_{20}-A_{21}-A_{22}-A_{23}$
$S-A_{12}-A_5-A_8-A_{16}-A_{19}$	A5	A8	A12	A16	A19
$A_4+A_7+A_{11}+A_{15}-A_{20}-A_{21}-A_{22}-A_{23}-A_1-A_2-A_3+A_5+A_8+A_{12}+A_{16}$	$S-A_4-A_5-A_{12}+A_{21}+A_{22}+A_{23}-A_{17}-A_7-A_8-A_{15}-A_{16}-A_{13}-A_9-A_{11}+A_1+A_2+A_3$	A9	A13	A17	A20
A1	$S+A_{16}+A_{13}+A_6-A_{19}+A_3-A_1-A_{18}-A_{14}-2*A_{21}-A_{22}-A_{23}-A_{20}$	$A_{21}+A_{22}+A_{23}+A_{20}-A_{16}-A_{13}-A_6+A_{19}-A_3$	A14	A18	A21
A2	A6	$S+A_{16}+A_{13}-A_9+A_6-A_{19}+A_3-A_{10}-A_{21}-A_{22}-A_{23}-A_{20}-A_7-A_8$	$A_5+A_{14}+A_{10}+2*A_{21}+A_{22}+3*A_{23}+2*A_{20}-A_4+A_8-A_{15}-A_{16}-A_{13}+2*A_9-2*A_6+2*A_{19}-A_{11}-A_2-A_3-S$	$S-A_{19}-A_{20}-A_{21}-A_{22}-2*A_{23}+A_4+A_7+A_{11}+A_{15}-A_5-A_9-A_{14}$	A22
A3	$A_{12}+A_{18}+A_{14}+A_{21}+A_{20}+A_{17}+A_7+A_8+A_1+5+A_9-2*A_6+A_{19}+A_{11}-A_2-2*A_3-S$	A10	$2*S+A_4-A_8-A_{15}+A_{16}-2*A_9+2*A_6-2*A_{19}+A_2+A_3-A_{12}-A_5-2*A_{14}-A_{10}-2*A_{21}-A_{22}-3*A_{23}-2*A_{20}$	$A_{19}+A_{20}+A_{21}+A_{22}+2*A_{23}-A_4-A_7-A_{11}+A_5+A_9+A_{14}-2*A_{15}-A_{16}-A_{17}-A_{18}$	A23

### Details:

It is a normal magic square of order 6. The letter S represent the magic sum of order 6.

Below are two examples based on the Result 3.2.

**Example 3.3.** Let's consider following two examples:

2a	mgc							150
	78	14	17	21	25	-5		150
	40	15	18	22	26	29		150
	-4	55	19	23	27	30		150
	11	-21	77	24	28	31		150
	12	16	-1	110	-19	32		150
	13	71	20	-50	63	33		150
	150	150	150	150	150	150		150

2b	mgc							175
	78	14	17	21	25	20		175
	65	15	18	22	26	29		175
	-4	80	19	23	27	30		175
	11	4	77	24	28	31		175
	12	16	24	85	6	32		175
	13	46	20	0	63	33		175
	175	175	175	175	175	175		175

## 3.2 Reduced Entries Pandiagonal Square of Order 6

**Result 3.3.** Let's consider a following magic square of order 6 with reduced entries:

S-A1-A2-A3-A4-A5	A1	A2	A3	A4	A5
A6	A7	A8	$S-2*A4-A12-A11-2*A3-2*A5-A2+A6+2*A7-A13+2*A16-A1+A9$	$2*S/3-A15-A7-A16$	$A15-2*S/3+2*A4-A8+A12-2*A6-A16+A11+2*A3+2*A5+A2-2*A7+A13+A1-A9$
A9	$5*S/6+A5+A3+A4-A8-A6-2*A7-A10-A16-A9$	A10	A11	$A15+A11+3*A3+3*A4+4*A5+A2-A14-A8+2*A12-2*A6-3*A7-A10+2*A13-3*A16+2*A1-2*A9-5*S/6$	$S-A15-2*A11-4*A3-4*A4-5*A5-A2+A14+2*A8-2*A12+3*A6+5*A7+A10-2*A13+4*A16-2*A1+2*A9$
$A11-S/3+2*A5+A2+A13+A3+A4-A14-A6-A7-A16+A1-A9$	A12	A13	$A7+A16-A11+A14+A6-A13+A9-A3-A5$	$2*S/3-A4-A12-A1$	$2*S/3-A5-A2-A13$
A14	A15	$4*S/3-A15-3*A5-2*A13-3*A4+A8-2*A12+2*A6+3*A7-A11-2*A3-A2+2*A16-A1+A9$	$A11+2*A3+2*A4+2*A5+A2-A14+A12-2*A6-2*A7+A13-2*A16+A1-A9-S/3$	A16	$A4+A5-A8+A12-A7+A13-A16$
$S/3-A11-A5-A13+A7+A16$	$S/6+A8-A12+A6+A7+A10+A16-A1+A9-A15-A5-A3-A4$	$A15-S/3+3*A5+3*A4-2*A8+2*A12-2*A6-3*A7+A11+2*A3-A10+A13-2*A16+A1-A9$	$S/3+A5+A13-A7-A16-A9$	$S/2-A2-3*A3-3*A4+A14+A8-A12+A10+3*A16-A1-4*A5+2*A6+4*A7-2*A13+2*A9-A11$	$A11+2*A3+A4+2*A5+A2-A14-A6-2*A7-A10+A13-2*A16+A1-A9$

### Details:

It is a **pandiagonal** magic square of order 6. The letter S represent the magic sum of order 6. In order to get non-decimal entries, the magic sum should be multiple of 6. If the entries in a magic square of order 6 are sequential numbers, then it is impossible to get **pandiagonal** magic square. In this case the entries are non sequential type. See the examples below.

**Example 3.4.** Let's consider following two examples representing **pandiagonal** magic squares of order 6

3a	pan	60	60	60	60	60	60		3b	pan	90	90	90	90	90	90
60	-5	11	12	13	14	15	60		90	-15	15	18	21	24	27	90
60	16	17	18	8	-28	29	60		90	30	33	36	24	-90	57	90
60	19	-41	20	21	0	41	60		90	39	-126	42	45	-33	123	90
60	2	22	23	30	-7	-10	60		90	-18	48	51	72	-27	-36	90
60	24	25	-20	-8	26	13	60		90	54	57	-54	-48	60	21	90
	4	26	7	-4	55	-28	60			0	63	-3	-24	156	-102	90
	60	60	60	60	60	60	60			90	90	90	90	90	90	90

The above example is without any division. The result below give another way to write a **pandiagonal** magic square of order 6, where we have four equal sums **semi-magic** squares of order 3.



**Result 3.4.** Let's consider a following magic square of order 6 with reduced entries:

$2*S/3-A6$	$A6+A7-S/3$	$2*S/3-A7$	$2*S/3-A2$	$A2+A3-S/3$	$2*S/3-A3$
$A6+A8-S/3$	$S+A1-A6-A7-A8$	$S/3+A7-A1$	$A2+A4-S/3$	$5*S/3-A3-A2-A4-A5$	$A3+A5-S/3$
$2*S/3-A8$	$S/3+A8-A1$	$A1$	$2*S/3-A4$	$A4+A5-S/3$	$2*S/3-A5$
$A2$	$S-A2-A3$	$A3$	$A6$	$S-A6-A7$	$A7$
$S-A2-A4$	$A2+A3+A4+A5-S$	$S-A3-A5$	$S-A6-A8$	$A6+A7+A8-A1-S/3$	$S/3-A7+A1$
$A4$	$S-A4-A5$	$A5$	$A8$	$S/3-A8+A1$	$2*S/3-A1$

**Details:**

It is a **pandiagonal** magic square of order 6. The letter  $S$  represent the **semi-magic** sum of order 3. In order to get **non-decimal** entries, the magic sum of order 6 should be multiple of 6. If the entries in a magic square of order 6 are sequential numbers, then it is impossible to get **pandiagonal** magic square. In this case the entries are non sequential. See the example below.

**Example 3.5.** Let's consider a following example representing **pandiagonal** magic squares of order 6

	pan	120	120	120	120	120	120
120	14	35	11	26	11	23	120
120	38	-16	38	14	26	20	120
120	8	41	11	20	23	17	120
120	14	29	17	26	5	29	120
120	26	14	20	2	56	2	120
	20	17	23	32	-1	29	120
	120	120	120	120	120	120	120

The above **pandiagonal** magic square is divided in four equal parts of **semi-magic** squares of order 3. See the figure below.



semi				3	semi				69
14	35	11	60		26	11	23	60	
38	-16	38	60		14	26	20	60	
8	41	11	60		20	23	17	60	
60	60	60	9		60	60	60	69	
semi				51	semi				117
14	29	17	60		26	5	29	60	
26	14	20	60		2	56	2	60	
20	17	23	60		32	-1	29	60	
60	60	60	51		60	60	60	111	

**Remark 1.** The above magic square is constructed with equal sums *semi-magic* squares of order 3. The work with equal sums magic square of order 3 shall be done elsewhere.

### 3.3 Reduced Entries Semi-Magic Squares of Order 6

Below is a result for **reduced entries semi-magic** squares based on Example 3.1.

**Result 3.5.** Let's consider following *semi-magic* square of order 6 with *reduced entries*:

$4*A_{15}-6*A_7+A_{12}$	$A_{15}+2*A_{13}+2*A_{14}-4*A_{12}-A_7$	$2*A_{12}+9*A_7-6*A_{15}-A_{13}-A_{14}$	$A_{15}-A_{13}-A_7$	$A_{15}-A_{14}-A_7$	$A_{12}$
$4*A_7-3*A_{15}+A_9+A_{10}+A_{11}$	$A_1$	$A_4+A_7-A_6-A_1$	$A_6-A_3-A_4$	$A_3$	$4*A_{15}-A_9-A_{10}-A_{11}-5*A_7$
$A_{15}-A_9-A_7$	$A_6$	$A_4$	$A_5$	$A_7-A_4-A_5-A_6$	$A_9$
$A_{15}-A_{10}-A_7$	$A_2+A_3-A_6$	$A_1+A_2-A_5$	$A_7-A_1-A_2-A_4$	$A_4+A_5+A_6-A_2-A_3$	$A_{10}$
$A_{15}-A_{11}-A_7$	$A_7-A_1-A_2-A_3$	$A_5-A_2-2*A_4+A_6$	$A_1+A_2+A_3+2*A_4-A_5-A_6$	$A_2$	$A_{11}$
$5*A_7-A_{12}-3*A_{15}$	$4*A_{12}-2*A_{13}-2*A_{14}$	$7*A_{15}+A_{13}+A_{14}-4-2*A_{12}-10*A_7$	$A_{13}$	$A_{14}$	$5*A_7-A_{12}-3*A_{15}$

*Details:*

It is a **single-digit bordered semi-magic** square of order 6 embedded magic square of orders 4. It is *semi-magic* only at one diagonal.  
 In order to bring it as a magic square we need the following condition:

$$M = \frac{2}{3} \times S, \quad (1)$$

where  $M$  and  $S$  are magic sums of magic squares of orders 4 and 6 respectively.

Below are two examples based on the Result 3.5. First example is of **semi-magic** squares and the second example as a magic square obtained by applying the conditions given in (1)

**Example 3.6.** Let's consider a following **semi-magic** square based on the Result 3.5:

s1	semi	M=2*S/3					180
	24	68	21	3	0	44	160
	74	11	93	-11	17	-24	160
	15	26	20	23	41	35	160
	12	5	2	65	38	38	160
	9	68	-5	33	14	41	160
	26	-18	29	47	50	26	160
	160	160	160	160	160	160	160

The **magic** and **semi-magic** sums of above magic square are  $M_{4 \times 4} = 110$  and  $S_{SM_{6 \times 6}} = 160$ .

**Example 3.7.** Let's consider a following magic square with **even** number magic sum based on the Result 3.5:

s2	mgc	M=2*S/3					150
	44	68	-9	3	0	44	150
	64	11	83	-11	17	-14	150
	15	26	20	23	31	35	150
	12	5	2	55	38	38	150
	9	58	-5	33	14	41	150
	6	-18	59	47	50	6	150
	150	150	150	150	150	150	150

The **magic** sums of above magic square are  $M_{4 \times 4} = 100$  and  $S_{6 \times 6} = 150$ . It is a magic square as it satisfies the conditions given in (1), i.e.,  $M = \frac{2}{3} \times S$ .

## 4 Magic Squares of Order 8

**Example 4.1.** Let's consider following four magic square of order 8

	pan	260	260	260	260	260	260	260	260
260	7	60	1	62	15	52	9	54	260
260	2	61	8	59	10	53	16	51	260
260	64	3	58	5	56	11	50	13	260
260	57	6	63	4	49	14	55	12	260
260	23	44	17	46	31	36	25	38	260
260	18	45	24	43	26	37	32	35	260
260	48	19	42	21	40	27	34	29	260
260	41	22	47	20	33	30	39	28	260
	260	260	260	260	260	260	260	260	260

mgc									260
31	36	25	38	43	22	13	52		260
26	37	32	35	23	42	1	64		260
40	27	34	29	41	24	54	11		260
33	30	39	28	49	16	9	56		260
18	17	44	50	20	46	7	58		260
47	48	21	15	19	45	53	12		260
10	8	62	51	6	61	60	2		260
55	57	3	14	59	4	63	5		260
260	260	260	260	260	260	260	260		260

mgc									260
8	2	62	64	51	13	53	7		260
5	46	44	17	50	18	20	60		260
6	16	31	36	25	38	49	59		260
11	22	26	37	32	35	43	54		260
61	24	40	27	34	29	41	4		260
56	42	33	30	39	28	23	9		260
55	45	21	48	15	47	19	10		260
58	63	3	1	14	52	12	57		260
260	260	260	260	260	260	260	260		260

mgc									260
3	1	63	61	60	58	8	6		260
62	64	2	4	5	7	57	59		260
17	48	31	36	25	38	21	44		260
20	45	26	37	32	35	24	41		260
47	18	40	27	34	29	43	22		260
46	19	33	30	39	28	42	23		260
53	55	9	11	16	14	50	52		260
12	10	56	54	49	51	15	13		260
260	260	260	260	260	260	260	260		260

The first magic square is **pandiagonal** with four equal sum **pandiagonals** magic squares of order 4. The second magic square **cornered** magic square. The third magic square is **single-digit bordered** magic squares. The fourth magic square is **double-digit bordered** magic square. The internal magic squares are of orders 4 and 6. In case of forth example, there is only one internal magic square of order 4. More details on different kinds of magic squares of order 8 refer author's work [11, 16, 25, 27]. Based on last three magic squares we shall construct **reduced entries** magic squares of order 8.

## 4.1 Reduced Entries Magic Squares of Order 8

Below are three magic squares of reduced entries constructed based on the Example 4.1.

**Result 4.1.** Let's consider a following magic squares of order 8 with reduced entries:

A18	A19	$2*(T-M)/2-A15-A16-A17$	A15	A16	A17	$2*M-T+A18-A19+A23$	$T-M-A23-2*A18$
$2*A13+A24+A14+2*A18-T+M-A19$	$T-M-2*A18-A13$	$A15+A16+A17-(T-M)/2$	$(T-M)/2-A15$	$(T-M)/2-A16$	$(T-M)/2-A17$	$T-M-A23-A13-A24-A18+A19$	$2*M-T+A18+A23-A14$
$T-M-A7-A8-A9$	$A7+A8+A9-(T-M)/2$	A1	$M+A4-A6-A1$	$A6-A3-A4$	A3	$A10+A11+A12-(T-M)/2$	$T-M-A10-A11-A12$
A7	$(T-M)/2-A7$	A6	A4	A5	$M-A4-A5-A6$	$(T-M)/2-A10$	A10
A8	$(T-M)/2-A8$	$A2+A3-A6$	$A1+A2-A5$	$M-A1-A2-A4$	$A4+A5+A6-A2-A3$	$(T-M)/2-A11$	A11
A9	$(T-M)/2-A9$	$M-A1-A2-A3$	$A5-A2-2*A4+A6$	$A1+A2+A3+2*A4-A5-A6$	A2	$(T-M)/2-A12$	A12
$T-2*A13-A14+A19-3*A18-A23-A24$	$A23+3*A18+A24-A19+A13-T+M$	$A20+A21+A22-(T-M)/2$	$(T-M)/2-A20$	$(T-M)/2-A21$	$(T-M)/2-A22$	A13	A14
A23	$M-A23-A18-A24$	$T-M-A20-A21-A22$	A20	A21	A22	A24	A18

*Details:*

It is a **double-digit bordered** magic square of order 8 embedded a magic square of order 4. The four magic rectangles of orders  $2 \times 4$  are of equal width and length. The letters M and T represents the magic squares of orders 4 and 8 respectively. In order to avoid decimal entries, the magic sums M and T should be of same type, i.e., either **even** or **odd** numbers.

**Example 4.2.** Let's consider following two examples:

1a	mgc									200
	45	47	-43	39	41	43	93	-65	200	200
	127	-45	83	1	-1	-3	-65	103	200	200
	5	35	11	105	-11	15	53	-13	200	200
	23	17	21	17	19	63	11	29	200	200
	25	15	7	5	79	29	9	31	200	200
	27	13	81	-7	33	13	7	33	200	200
	-107	155	113	-9	-11	-13	35	37	200	200
	55	-37	-73	49	51	53	57	45	200	200
	200	200	200	200	200	200	200	200	200	200

1b	mgc									221
	45	47	-43	39	41	43	114	-65	221	221
	127	-45	83	1	-1	-3	-65	124	221	221
	5	35	11	126	-11	15	53	-13	221	221
	23	17	21	17	19	84	11	29	221	221
	25	15	7	5	100	29	9	31	221	221
	27	13	102	-7	33	13	7	33	221	221
	-86	155	113	-9	-11	-13	35	37	221	221
	55	-16	-73	49	51	53	57	45	221	221
	221	221	221	221	221	221	221	221	221	221

The magic sums of above two examples are as follows:

**First Example**

$$M_{4 \times 4} = 120$$

$$T_{8 \times 8} = 200$$

**Second Example**

$$M_{4 \times 4} = 141$$

$$T_{8 \times 8} = 221$$

Above two examples are considered with magic sums as **even** and **odd** numbers. The magic rectangles sums are:

**First Example**

$$MR_{2 \times 4} = 40 \times 80$$

**Second Example**

$$MR_{2 \times 4} = 40 \times 80.$$

**Result 4.2.** Let's consider following *cornered* magic squares of order 8 with reduced entries:

A1	A4+M-A6-A1	A6-A3-A4	A3	S-M-A10	A10	a18	T-S-A18
A6	A4	A5	M-A4-A5-A6	S-M-A11	A11	a19	T-S-A19
A2+A3-A6	A1+A2-A5	M-A1-A2-A4	A4+A5+A6-A2-A3	M-S+A10 +A11+A12	2*(S-M)- A10-A11-A12	A20	T-S-A20
M-A1-A2-A3	A5-A2-2*A4+A6	A1+A2+A3 +2*A4-A5-A6	A2	S-M-A12	A12	3*(T-S)-A18-A19- A20-A21-A22	2*(S-T)+A18+A19 +A20+A21+A22
2*S-3*M+A1- A8+3*A4- A11+A10	S-M-A8	3*M-2*S-A1 +2*A8-3*A4 +A11-A10+A9	S-M-A9	(S-M)/2	(5*M-3*S)/2	A21	T-S-A21
2*M-S-A1+ A8-3*A4+ A11-A10	A8	3*S-4*M+A1- 2*A8 +3*A4- A11+A10-A9	A9	(5*M-3*S)/2	(S-M)/2	A22	T-S-A22
A14	2*A11-T-5*S-A1 +2*A9+2*A8+2*A12+A 14-3*A4+A18-A19+8*M	A15	4*T+2*S-2*A14+A1- 2*A11-2*A9-2*A8- 2*A12+3*A4-A18+A19- 8*M-A15-A16-A17	A16	A17	(T-S)/2	(7*S-5*T)/2
T-S-A14	2*T+4*S+A1-2*A11- 2*A9-2*A8-2*A12- A14+3*A4-A18+A19- 8*M	T-S-A15	2*A14-3*S-3*T-A1 +2*A11+2*A9+2*A8 +2*A12-3*A4+A18-A19 +8*M+A15+A16+A17	T-S-A16	T-S-A17	(7*S-5*T)/2	(T-S)/2

**Details:**

It is a **cornered** magic square of order. The magic rectangles of orders  $2 \times 4$  and  $2 \times 6$  are of equal width and length in each case. The letters M, S and T represents the magic squares of orders 4, 6 and 8 respectively. In order to avoid decimal entries, the magic sums M, S and T should be of same type, i.e., either **even** or **odd** numbers.

**Example 4.3.** Let's consider following two examples:



2a	mgc									220
	21	87	-21	23	0	30	43	37	220	220
	26	24	25	35	63	-33	50	30	220	220
	19	18	43	30	-1	31	24	56	220	220
	44	-19	63	22	-2	32	42	38	220	220
	2	-74	131	1	15	65	41	39	220	220
	28	104	-101	29	65	15	40	40	220	220
	56	180	-131	46	45	44	40	-60	220	220
	24	-100	211	34	35	36	-60	40	220	220
	220	220	220	220	220	220	220	220	220	220

2b	mgc									235
	21	88	-21	23	10	30	47	37	235	235
	26	24	25	36	53	-13	23	61	235	235
	19	18	44	30	9	31	47	37	235	235
	45	-19	63	22	8	32	46	38	235	235
	12	-53	110	11	20	51	45	39	235	235
	28	93	-70	29	51	20	44	40	235	235
	90	201	-186	50	49	48	42	-59	235	235
	-6	-117	270	34	35	36	-59	42	235	235
	235	235	235	235	235	235	235	235	235	235

The magic sums of above two examples are as follows:

#### First Example

$$M_{4 \times 4} = 110$$

$$S_{6 \times 6} = 140$$

$$T_{8 \times 8} = 220$$

#### Second Example

$$M_{4 \times 4} = 111$$

$$S_{6 \times 6} = 151$$

$$T_{8 \times 8} = 235$$

Above two examples are considered with magic sums as **even** and **odd** numbers. The magic rectangles sums are:

#### First Example

$$MR_{2 \times 4} = 30 \times 60$$

$$MR_{2 \times 6} = 80 \times 240$$

#### Second Example

$$MR_{2 \times 4} = 40 \times 80$$

$$MR_{2 \times 6} = 84 \times 252$$

**Result 4.3.** Let's consider following *cornered* magic squares of order 8 with reduced entries:



$4*S-6*M+A12$	$S+2*A13+2*A14-4*A12-M$	$2*A12+9*M-6*S-A13-A14$	$S-A13-M$	$S-A14-M$	$A12$	$T-S-A16$	$A16$
$4*M-3*S+A9+A10+A11$	$A1$	$A4+M-A6-A1$	$A6-A3-A4$	$A3$	$4*S-A9-A10-A11-5*M$	$T-S-A16-A1-A13-A9-A14+2*A12-7*S-3*A4+10*M$	$A16+A1+A13+A9+A14-2*A12+7*S+3*A4-10*M$
$S-A9-M$	$A6$	$A4$	$A5$	$M-A4-A5-A6$	$A9$	$2*A16+A18+A19+A1-2*T+9*S+A13+A9+A14-2*A12+3*A4-10*M+A17$	$3*T-10*S-A18-A19-2*A16-A1-A13-A9-A14+2*A12-3*A4+10*M-A17$
$S-A10-M$	$A2+A3-A6$	$A1+A2-A5$	$M-A1-A2-A4$	$A4+A5+A6-A2-A3$	$A10$	$T-S-A17$	$A17$
$S-A11-M$	$M-A1-A2-A3$	$A5-A2-2*A4+A6$	$A1+A2+A3+2*A4-A5-A6$	$A2$	$A11$	$T-S-A18$	$A18$
$5*M-A12-3*S$	$4*A12-2*A13-2*A14$	$7*S+A13+A14-2*A12-10*M$	$A13$	$A14$	$5*M-A12-3*S$	$T-S-A19$	$A19$
$T-S+A13+A9+A14-2*A12+2*A1+7*S+6*A4-11*M$	$8*S+A13+A9+A14-2*A12+2*A1+6*A4-11*M$	$A21+A20+A22-T-2*A13-2*A9-2*A14+4*A12-4*A1-14*S-12*A4+22*M$	$T-S-A20$	$T-S-A21$	$T-S-A22$	$(T-S)/2$	$(7*S-5*T)/2$
$11*M-A13-A9-A14+2*A12-2*A1-7*S-6*A4$	$T-9*S-A13-A9-A14+2*A12-2*A1-6*A4+11*M$	$2*T+13*S+2*A13+2*A9+2*A14-4*A12+4*A1+12*A4-22*M-A21-A20-A22$	$A20$	$A21$	$A22$	$(7*S-5*T)/2$	$(T-S)/2$

### Details:

It is a **cornered** magic square of order embedded with a **single-digit bordered** magic square of order 6. It has magic square of order 4 as an inner part. The magic rectangles of orders  $2 \times 6$  are of equal width and length. The letters M, S and T represents the magic squares of orders 4, 6 and 8 respectively. In order to avoid decimal entries, the magic sums M, S and T should be of same type, i.e., either **even** or **odd** numbers.

**Example 4.4.** Let's consider following two examples:

3a	mgc								230
	24	68	21	3	0	44	14	56	230
	74	11	93	-11	17	-24	-121	191	230
	15	26	20	23	41	35	293	-223	230
	12	5	2	65	38	38	11	59	230
	9	68	-5	33	14	41	8	62	230
	26	-18	29	47	50	26	5	65	230
	166	256	-209	2	-1	-4	35	-15	230
	-96	-186	279	68	71	74	-15	35	230
	230	230	230	230	230	230	230	230	230

3b	mgc								231
	28	69	15	4	1	44	14	56	231
	71	11	93	-11	17	-20	-128	198	231
	16	26	20	23	41	35	300	-230	231
	13	5	2	65	38	38	11	59	231
	10	68	-5	33	14	41	8	62	231
	23	-18	36	47	50	23	5	65	231
	173	264	-224	2	-1	-4	35	-14	231
	-103	-194	294	68	71	74	-14	35	231
	231	231	231	231	231	231	231	231	231

The magic sums of above two examples are as follows:

**First Example**

$$M_{4 \times 4} = 110$$

$$S_{6 \times 6} = 160$$

$$T_{8 \times 8} = 230$$

**Second Example**

$$M_{4 \times 4} = 110$$

$$S_{6 \times 6} = 161$$

$$T_{8 \times 8} = 231$$

Above two examples are considered with magic sums as **even** and **odd** numbers. The magic rectangles sums are:

**First Example**

$$MR_{2 \times 6} = 70 \times 210$$

**Second Example**

$$MR_{2 \times 6} = 70 \times 210$$

**Result 4.4.** Let's consider following *cornered* magic squares of order 8 with reduced entries:

$A_{19}+A_{20}+A_{21}+A_{22}+A_{23}-A_4-A_7-A_{11}-A_{15}$	A4	A7	A11	A15	$S-A_{19}-A_{20}-A_{21}-A_{22}-A_{23}$	T-S-A29	A29
$S-A_{12}-A_5-A_8-A_{16}-A_{19}$	A5	A8	A12	A16	A19	T-S-A30	A30
$A_4+A_7+A_{11}+A_{15}-A_{20}-A_{21}-A_{22}-A_{23}-A_1-A_2-A_3+A_5+A_8+A_{12}+A_{16}$	$S-A_4-A_5-A_{12}+A_{21}+A_{22}+A_{23}-A_{17}-A_7-A_8-A_{15}-A_{16}-A_{13}-A_9-A_{11}+A_1+A_2+A_3$	A9	A13	A17	A20	T-S-A31	A31
A1	$S+A_{16}+A_{13}+A_6-A_{19}+A_3-A_1-A_{18}-A_{14}-2*A_{21}-A_{22}-A_{23}-A_{20}$	$A_{21}+A_{22}+A_{23}+A_{20}-A_{16}-A_{13}-A_6+A_{19}-A_3$	A14	A18	A21	T-S-A32	A32
A2	A6	$S+A_{16}+A_{13}-A_9+A_6-A_{19}+A_3-A_{10}-A_{21}-A_{22}-A_{23}-A_{20}-A_7-A_8$	$A_5+A_{14}+A_{10}+2*A_{21}+A_{22}+3*A_{23}+2*A_{20}-A_4+A_8-A_{15}-A_{16}-A_{13}+2*A_9-2*A_6+2*A_{19}-A_{11}-A_2-A_3-S$	$S-A_{19}-A_{20}-A_{21}-A_{22}-2*A_{23}+A_4+A_7+A_{11}+A_{15}-A_5-A_9-A_{14}$	A22	T-S-A33	A33
A3	$A_{12}+A_{18}+A_{14}+A_{21}+A_{20}+A_{17}+A_7+A_8+A_{15}+A_9-2*A_6+A_{19}+A_{11}-A_2-2*A_3-S$	A10	$2*S+A_4-A_8+A_{15}+A_{16}-2*A_9+2*A_6-2*A_{19}+A_2+A_3-A_{12}-A_5-2*A_{14}-A_{10}-2*A_{21}-A_{22}-3*A_{23}-2*A_{20}$	$A_{19}+A_{20}+A_{21}+A_{22}+2*A_{23}-A_4-A_7-A_{11}+A_5+A_9+A_{14}-2*A_{15}-A_{16}-A_{17}-A_{18}$	A23	$2*S-2*T+A_{29}+A_{30}+A_{31}+A_{32}+A_{33}$	$3*T-3*S-A_{29}-A_{30}-A_{31}-A_{32}-A_{33}$
T-S-A25	$A_{30}-A_{29}-2*A_9+A_{13}-A_{25}+2*S-2*A_{21}-A_{22}-3*A_{23}-A_{18}-2*A_{19}-3*A_{20}-A_{14}+A_{15}+A_{16}-2*A_{10}+A_2+A_4+A_{11}-A_5+2*A_6-A_8+A_3$	T-S-A26	T-S-A27	T-S-A28	$2*A_{25}+A_{28}+A_{27}+A_{26}-A_{30}+A_{29}+2*A_9-A_{13}-T-S+2*A_{21}+A_{22}+3*A_{23}+A_{18}+2*A_{19}+3*A_{20}+A_{14}-A_{15}-A_{16}+2*A_{10}-A_2-A_4-A_{11}+A_5-2*A_6+A_8-A_3$	(T-S)/2	(7*S-5*T)/2
A25	$A_{29}-A_{30}+2*A_9-A_{13}+T+A_{25}-A_{11}-3*S+2*A_{21}+A_{22}+3*A_{23}+A_{18}+2*A_{19}+3*A_{20}+A_{14}-A_{15}-A_{16}+2*A_{10}-A_2-A_4+A_5-2*A_6+A_8-A_3$	A26	A27	A28	$A_{30}-A_{29}-A_{28}-A_{27}-A_{26}-2*A_9+A_{13}+2*T-2*A_{25}-2*A_{21}-A_{22}-3*A_{23}-A_{18}-2*A_{19}-3*A_{20}-A_{14}+A_{15}+A_{16}-2*A_{10}+A_2+A_4+A_{11}-A_5+2*A_6-A_8+A_3$	(7*S-5*T)/2	(T-S)/2

### Details:

It is a **cornered** magic square of order embedded with a magic square of order 6. The magic rectangles of orders  $2 \times 6$  are of equal width and length. The letters S and T represents the magic squares of orders 6 and 8 respectively. In order to avoid decimal entries, the magic sums S and T should be of same type, i.e., either **even** or **odd** numbers.

**Example 4.5.** Let's consider following two examples:

4a	mgc									220
	212	20	29	41	53	-255	28	92	220	220
	-120	23	32	44	56	65	13	107	220	220
	-34	-75	35	47	59	68	253	-133	220	220
	11	-303	209	50	62	71	25	95	220	220
	14	26	-243	526	-297	74	22	98	220	220
	17	409	38	-608	167	77	19	101	220	220
	16	-683	925	37	34	31	60	-200	220	220
	104	803	-805	83	86	89	-200	60	220	220
	220	220	220	220	220	220	220	220	220	220

4b	mgc									251
	212	20	29	41	53	-234	38	92	251	251
	-99	23	32	44	56	65	23	107	251	251
	-34	-54	35	47	59	68	233	-103	251	251
	11	-282	209	50	62	71	35	95	251	251
	14	26	-222	505	-276	74	32	98	251	251
	17	388	38	-566	167	77	29	101	251	251
	26	-641	873	47	44	41	65	-204	251	251
	104	771	-743	83	86	89	-204	65	251	251
	251	251	251	251	251	251	251	251	251	251

The magic sums of above two examples are as follows:

**First Example**

$$S_{6 \times 6} = 100$$

$$T_{8 \times 8} = 220$$

**Second Example**

$$S_{6 \times 6} = 121$$

$$T_{8 \times 8} = 251$$

Above two examples are considered with magic sums as **even** and **odd** numbers. The magic rectangles sums are:

**First Example**

$$MR_{2 \times 6} = 120 \times 360$$

**Second Example**

$$MR_{2 \times 6} = 130 \times 390$$

## 4.2 Reduced Entries Striped Magic Square of Order 8

By striped magic squares we understand that the magic constructed only with equal width **magic rectangles**. For more details see the author's work [27]. The result below give reduced entries striped magic square of order 8. It also included magic squares of order 4.

**Result 4.5.** *Let's consider following **striped** square of order 8 with **reduced entries**:*

$A1-A3-A5-A6-A8+A7$	$A6+A8-2*m+A11+A1-A3-A5$	$2*m$	$2*m+2*A3+2*A5-A7-A11-2*A1$	$A5+2*A3-A7-A11+A10-A9$	$2*m-A9-A5-A10$	$A9-2*A3+A7+A11$	$A9$
$m+A3+A5+A6+A8-A7-A1$	$3*m+A3+A5-A6-A8-A11-A1$	$(-m)$	$A7-m+A11+2*A1-2*A3-2*A5$	$m+A9-A10+A11-A5-2*A3+A7$	$A9+A5+A10-m$	$m-A9+2*A3-A7-A11$	$m-A9$
$2*A3+2*A5-A7-A11-A1$	$A1$	$A6+A8+A11-A3-A5$	$2*m+A7-A3-A5-A6-A8$	$A5+2*A3-A7-A11$	$A5$	$2*m+A7+A11-A10-2*A3-2*A5$	$A10$
$A1+m-2*A3-2*A5+A7+A11$	$m-A1$	$m+A3+A5-A6-A8-A11$	$A3+A5+A6+A8-A7-m$	$m+A7+A11-A5-2*A3$	$m-A5$	$2*A3+2*A5-A7-A11+A10-m$	$m-A10$
$A3+A2-A7-A11+A4$	$2*m-A3-A2-A4$	$A7+A11-A3$	$A3$	$2*m-A6-A8-A11$	$A6+A8-A7$	$A7$	$A11$
$m+A7+A11-A4-A3-A2$	$A3+A2+A4-m$	$m+A3-A7-A11$	$m-A3$	$A6+A8+A11-m$	$m+A7-A6-A8$	$m-A7$	$m-A11$
$A2+2*A3-A7-A11$	$A2$	$2*m+A7+A11-2*A2-A4-2*A3$	$A4$	$A6+A11-A7$	$A6$	$A8$	$2*m+A7-2*A6-A8-A11$
$m+A7+A11-A2-2*A3$	$m-A2$	$2*A3-A7-A11+2*A2+A4-m$	$m-A4$	$m+A7-A11-A6$	$m-A6$	$m-A8$	$2*A6+A8+A11-A7-m$

### Details:

It is a **striped** magic square of order 8 where the magic rectangles of order  $2 \times 4$  are of equal width and length, i.e.,  $m \times 2m$ . In this case the magic sum is  $T = m \times 2m$ . It includes five magic squares of order 4 also. These are specified in an example.

**Example 4.6.** Let's consider following example:

[illegible]

The magic square sum is  $T = 160$  and each magic rectangle of order  $2 \times 4$  is with magic sum  $40 \times 80$ . It contains five magic squares as given below:

[illegible]

The first four magic squares are of each corner and the fifth one is from the middle part.



### 4.3 Reduced Entries Pandiagonal Magic Square of Order 8

Below is a **panmagic** square of order 8 having the four magic squares of equal magic sums.

**Result 4.6.** Let's consider following a *panmagic* square of order 8 with *reduced entries*:

$2*T-2*A4-2*A7-A12+A3+A6$	$3*A4+3*A7-A15-A2+A12-A3-A6-2*T$	$A2+A14-A16-4*A4-4*A7+A3+A6-A1+4*T+A15$	$A16+3*A4-3*T+3*A7-A3-A6+A1-A14$	$A3$	$2*T-A9-A4-A7-A5-A3$	$A5+A8-A16+A3+A6-A1+A9$	$A16+A4+A7-A3-A6+A1-A8-T$
$A2$	$A4+A7-A15-T$	$A1$	$2*T-A4-A7+A15-A1-A2$	$A5$	$T-A9-A4-A7$	$A16+2*A4-2*T+2*A7+A1-A10$	$A9-A4-A7+2*T-A16-A1+A10-A5$
$2*A4+2*A7-A13-T-A14+A16-A3-A6+A1-A2$	$4*T-3*A4-3*A7-A12+A3+A6-A13-A1$	$2*A4+2*A7+A12-A3-A6 +A13+A15-2*T$	$A2+A13+A14-A16-A4-A7+A3+A6-A15$	$2*A4-T-A8+A16+A7-A3-A6+A1-A5$	$A3-A4+2*T-A16-2*A7-A1+A10$	$A9+A7-A3$	$A8+A3+A6-A10+A5-A9-A4$
$A12+A13+A14-A16-A1$	$A1-A4-A7+A13+2*A15+A2$	$2*A4+2*A7-A12-A13-A14-T+A16-2*A15-A2$	$2*T-A4-A7-A13$	$2*T-2*A4+A8-A16-A7+A6-A1$	$A16+3*A4-4*T+4*A7+A1+2*A9+A5-A10$	$3*T-A8-3*A7-A6-2*A9+A10-A5-2*A4$	$A4$
$A6$	$A9+T-A11-A6$	$A11-A8-A9$	$A8$	$A12$	$A15+T-A17-A12$	$A17-A14-A15$	$A14$
$A11$	$A9$	$A10$	$T-A9-A10-A11$	$A17$	$A15$	$A16$	$T-A15-A16-A17$
$A7+A8-A11$	$A6+A7-A10$	$T-A6-A7-A9$	$A9+A10+A11-A7-A8$	$A13+A14-A17$	$A12+A13-A16$	$T-A12-A13-A15$	$A15+A16+A17-A13-A14$
$T-A6-A7-A8$	$A10-A7-2*A9+A11$	$A6+A7+A8+2*A9-A10-A11$	$A7$	$T-A12-A13-A14$	$A16-A13-2*A15+A17$	$A12+A13+A14+2*A15-A16-A17$	$A13$

*Details:*

It is a **pandiagonal** magic square of order 8, where the four blocks of order  $4 \times 4$  are of equal magic sums magic squares. See below an example.

**Example 4.7.** Let's consider following example:



	pan	200	200	200	200	200	200	200	200
200	57	-87	265	-135	21	30	88	-39	200
200	16	-114	11	187	31	-18	-25	112	200
200	-89	108	95	-14	-44	72	71	1	200
200	116	193	-271	62	92	16	-34	26	200
200	36	54	-36	46	66	24	-66	76	200
200	61	51	56	-68	91	81	86	-158	200
200	26	21	-28	81	56	51	-118	111	200
	-23	-26	108	41	-113	-56	198	71	200
	200	200	200	200	200	200	200	200	200

It is **pandiagonal** magic square of order 8 with four equal sums magic squares of order 4. The magic square sum is  $T_{8 \times 8} = 200$  and each magic square of order 4 is with magic sum  $S_{4 \times 4} = 100$ . See below:

mgc				100
57	-87	265	-135	100
16	-114	11	187	100
-89	108	95	-14	100
116	193	-271	62	100
100	100	100	100	100

mgc				100
21	30	88	-39	100
31	-18	-25	112	100
-44	72	71	1	100
92	16	-34	26	100
100	100	100	100	100

  

mgc				100
36	54	-36	46	100
61	51	56	-68	100
26	21	-28	81	100
-23	-26	108	41	100
100	100	100	100	100

mgc				100
66	24	-66	76	100
91	81	86	-158	100
56	51	-118	111	100
-113	-56	198	71	100
100	100	100	100	100

**Remark 2.** The above pandiagonal magic square of order 8 is obtained with four equal sums magic square of order 4. The similar kind of work with four equal sum **pandiagonal** magic squares of order 4 shall be done elsewhere.

#### 4.4 Reduced Entries Semi-Magic Squares of Order 8

Below are 2 results for **reduced entries semi-magic** squares based on Examples 4.1.

**Result 4.7.** Let's consider following *semi-magic* square of order 8 with *reduced entries*:

T-S-A17	$2*T+3*S+A19-A14-A20$	T-S-A14	$A18+2*A17-7*T+2*S+A16+A15+2*A14+A21+A22+A24+2*A20+A23$	T-S-A15	T-S-A16	T-S-A18	T-A21-A22-A24-A20-A23-A17-A19
T-S-A20	A1	$A4+M-A6-A1$	$A6-A3-A4$	A3	S-M-A10	A10	A20
T-S-A21	A6	A4	A5	$M-A4-A5-A6$	$A10+A11+A12-S+M$	$2*S-2*M-A10-A11-A12$	A21
T-S-A22	$A2+A3-A6$	$A1+A2-A5$	$M-A1-A2-A4$	$A4+A5+A6-A2-A3$	S-M-A11	A11	A22
T-S-A23	$M-A1-A2-A3$	$A5-A2-2*A4+A6$	$A1+A2+A3+2*A4-A5-A6$	A2	S-M-A12	A12	A23
T-S-A24	S-M-A8	$2*S-A8-A1-3*A4-2*A10-A11-A12-M$	$2*A8+A1+A9+3*A4+2*A10+A11+A12+M-2*S$	S-M-A9	$(S-M)/2$	$(5*M-3*S)/2$	A24
T-S-A19	A8	$A8+A1+3*A4+2*A10+A11+A12-S$	$3*S-2*A8-A1-A9-3*A4-2*A10-A11-A12-2*M$	A9	$(5*M-3*S)/2$	$(S-M)/2$	A19
$A17+A19+7*S+A21+A22+A24+A20+A23-6*T$	$A14+A20-T-A19-4*S$	A14	$8*T-A18-2*A17-3*S-A16-A15-2*A14-A21-A22-A24-2*A20-A23$	A15	A16	A18	A17

**Details:**

It is a **single-digit bordered semi-magic** square of order 8 embedded with **cornered** magic squares of orders 6. It is **semi-magic** only at one diagonal. In order to bring it as a magic square we need the following condition:

$$S = \frac{3}{4} \times T, \quad (2)$$

where  $S$  and  $T$  are magic sums of magic squares of orders 6 and 8 respectively.

Below are two examples based on the Result 4.7. First example is of **semi-magic** squares and the second example as a magic square obtained by applying the conditions given in (2)

**Example 4.8.** Let's consider a following **semi-magic** square based on the Result 4.7:

1a	semi	S=3*T/4							230
	-3	851	3	-421	1	-1	-5	-215	210
	-9	21	85	-21	25	11	39	59	210
	-11	31	27	29	23	73	-23	61	210
	-13	17	15	39	39	9	41	63	210
	-15	41	-17	63	23	7	43	65	210
	-17	15	-89	161	13	25	35	67	210
	-7	35	139	-111	37	35	25	57	210
	285	-801	47	471	49	51	55	53	210
	210	210	210	210	210	210	210	210	210

The **magic** and **semi-magic** sums of above magic square are  $M_{4 \times 4} = 110$ ,  $S_{6 \times 6} = 160$  and  $T_{Sm_{8 \times 8}} = 210$ . There are two magic rectangles with equal magic sums  $MR_{2 \times 4} = 50 \times 100$ .

**Example 4.9.** Let's consider a following magic square with **even** number magic sum based on the Result 4.7:

1b	mgc	S=3*T/4							240
	7	971	13	-591	11	9	5	-185	240
	1	21	85	-21	25	31	39	59	240
	-1	31	27	29	23	53	17	61	240
	-3	17	15	39	39	29	41	63	240
	-5	41	-17	63	23	27	43	65	240
	-7	35	-49	121	33	35	5	67	240
	3	35	119	-51	37	5	35	57	240
	245	-911	47	651	49	51	55	53	240
	240	240	240	240	240	240	240	240	240

The **magic** and **semi-magic** sums of above magic square are  $M_{4 \times 4} = 110$ ,  $S_{6 \times 6} = 180$  and  $T_{8 \times 8} = 240$ . It is a magic square as it satisfies the conditions given in (2), i.e.,  $T = \frac{3}{4} \times T$ . Moreover, the magic rectangles sums is  $MR_{2 \times 4} = 70 \times 140$ .

**Result 4.8.** Let's consider following *semi-magic* square of order 8 with *reduced entries*:

T-S-A27	$2*T+3*S+A17-2*A14-A24-2*A13-2*A8-5*M-3*A4-A18-2*A15$	T-S-A24	$A28+2*A27+2*A14-7*T+2*S+A26+A25+2*A24+2*A13+2*A8+5*M+3*A4+A19+A20+A22+2*A18+2*A15+A21$	T-S-A25	T-S-A26	T-S-A28	T-A19-A20-A22-A18-A21-A27-A17
T-S-A18	S-M-A15	S-A13-M	$2*A11-3*A4-A13-A8+A9+M$	$3*A4-3*S+2*A13+A14+2*A8+A9-A10+2*A15+2*M$	S-A14-M	S-A8-A9-A10-A11-A15	A18
T-S-A19	S-M-A9	A1	A4+M-A6-A1	A6-A3-A4	A3	A9	A19
T-S-A20	S-M-A10	A6	A4	A5	M-A4-A5-A6	A10	A20
T-S-A21	S-M-A11	A2+A3-A6	A1+A2-A5	M-A1-A2-A4	A4+A5+A6-A2-A3	A11	A21
T-S-A22	S-M-A8	M-A1-A2-A3	A5-A2-2*A4+A6	A1+A2+A3+2*A4-A5-A6	A2	A8	A22
T-S-A17	$A8-4*S+A9+A10+A11+A15+5*M$	A13	$S-2*A11+3*A4+A13+A8-A9-2*M$	$4*S+A11-3*A4-2*A13-A14-2*A8-A10-2*A15-3*M$	A14	A15	A17
$A27+A17+7*S+A19+A20+A22+A18+A21-6*T$	$A24+2*A13+2*A8+5*M+3*A4+A18+2*A15-T-A17+2*A14-4*S$	A24	$8*T-A28-2*A27-2*A14-3*S-A26-A25-2*A24-2*A13-2*A8-5*M-3*A4-A19-A20-A22-2*A18-2*A15-A21$	A25	A26	A28	A27

*Details:*

It is a *single-digit bordered semi-magic* square of order 8 embedded with *single-digit bordered semi-magic* square of order 6. These are *semi-magic* only at one diagonal. In order to bring it as a magic square we need some conditions. It is as given below

$$M = \frac{2}{3} \times S \quad \text{and} \quad S = \frac{3}{4} \times T, \quad (3)$$

where  $M$ ,  $S$  and  $T$  are magic sums of magic squares of orders 4, 6 and 8 respectively.

Below are two examples based on the Result 4.8. First example is of **semi-magic** square and the second example as a magic square obtained by applying the conditions given in (3)

**Example 4.10.** Let's consider a following *semi-magic* square based on the Result 4.7:

2a	semi	S=3*T/4 and M=2*S/3							170
	35	291	38	-265	37	36	34	64	270
	43	56	58	99	-168	57	98	27	270
	42	61	11	107	-11	13	19	28	270
	41	60	16	14	15	75	20	29	270
	40	59	9	8	83	20	21	30	270
	39	62	84	-9	33	12	18	31	270
	44	-98	22	-19	248	23	24	26	270
	-14	-221	32	335	33	34	36	35	270
	270	270	270	270	270	270	270	270	270

The **magic** and **semi-magic** sums of above magic square are  $M_{4 \times 4} = 120$ ,  $S_{Sm_{6 \times 6}} = 200$  and  $L_{Sm_{8 \times 8}} = 270$ .

**Example 4.11.** Let's consider a following magic square with *even* number magic sum based on the Result 4.8:

2b	mgc	S=3*T/4 and M=2*S/3							220
	20	136	23	-35	22	21	19	14	220
	28	31	33	89	-83	32	63	27	220
	27	36	11	97	-11	13	19	28	220
	26	35	16	14	15	65	20	29	220
	25	34	9	8	73	20	21	30	220
	24	37	74	-9	33	12	18	31	220
	29	-8	22	-34	138	23	24	26	220
	41	-81	32	90	33	34	36	35	220
	220	220	220	220	220	220	220	220	220

The **magic** and **semi-magic** sums of above magic square are  $M_{4 \times 4} = 110$ ,  $S_{6 \times 6} = 165$  and  $T_{8 \times 8} = 220$ . It is a magic square as it satisfies the conditions given in (3).

**Result 4.9.** Let's consider following *semi-magic* square of order 8 with *reduced entries*:



T-S-A28	$2*T+3*S+A30-A25-A31$	T-S-A25	$A29+2*A28-7*T+2*S+A27+A26+2*A25+A32+A33+A35+2*A31+A34$	T-S-A26	T-S-A27	T-S-A29	T-A32-A33-A35-A31-A34-A28-A30
T-S-A31	$A19+A20+A21+A22+A23-A4-A7-A11-A15$	A4	A7	A11	A15	S-A19-A20-A21-A22-A23	A31
T-S-A32	S-A12-A5-A8-A16-A19	A5	A8	A12	A16	A19	A32
T-S-A33	$A4+A7+A11+A15-A20-A21-A22-A23-A1-A2-A3+A5+A8+A12+A16$	$S-A4-A5-A12+A21+A22+A23-A17-A7-A8-A15-A16-A13-A9-A11+A1+A2+A3$	A9	A13	A17	A20	A33
T-S-A34	A1	$S+A16+A13+A6-A19+A3-A1-A18-A14-2*A21-A22-A23-A20$	$A21+A22+A23+A20-A16-A13-A6+A19-A3$	A14	A18	A21	A34
T-S-A35	A2	A6	$S+A16+A13-A9+A6-A19+A3-A10-A21-A22-A23-A20-A7-A8$	$A5+A14+A10+2*A21+A22+3*A23+2*A20-A4+A8-A15-A16-A13+2*A9-2*A6+2*A19-A11-A2-A3-S$	$S-A19-A20-A21-A22-2*A23+A4+A7+A11+A15-A5-A9-A14$	A22	A35
T-S-A30	A3	$A12+A18+A14+A21+A20+A17+A7+A8+A15+A9-2*A6+A19+A11-A2-2*A3-S$	A10	$2*S+A4-A8+A15+A16-2*A9+2*A6-2*A19+A2+A3-A12-A5-2*A14-A10-2*A21-A22-3*A23-2*A20$	$A19+A20+A21+A22+2*A23-A4-A7-A11+A5+A9+A14-2*A15-A16-A17-A18$	A23	A30
$A28+A30+7*S+A32+A33+A35+A31+A34-6*T$	$A25+A31-T-A30-4*S$	A25	$8*T-A29-2*A28-3*S-A27-A26-2*A25-A32-A33-A35-2*A31-A34$	A26	A27	A29	A28

### Details:

It is a **single-digit bordered semi-magic** square of order 8 embedded with a magic square of order 6. These are **semi-magic** only at one diagonal. In order to bring it as a magic square we need a condition given in (2), where S and T are magic sums of magic squares of orders 6 and 8 respectively.

Below are two examples based on the Result 4.9. First example is of **semi-magic** square and the second example as a magic square obtained by applying the condition given in (2).

**Example 4.12.** Let's consider a following **semi-magic** square based on the Result 4.9:

3a	semi								280
	2	844	5	-566	4	3	1	-93	200
	-1	78	14	17	21	25	5	41	200
	-2	50	15	18	22	26	29	42	200
	-3	-4	65	19	23	27	30	43	200
	-4	11	-11	77	24	28	31	44	200
	-5	12	16	9	100	-9	32	45	200
	0	13	61	20	-30	63	33	40	200
	213	-804	35	606	36	37	39	38	200
	200	200	200	200	200	200	200	200	200

The **magic** and **semi-magic** sums of above magic square are  $S_{6 \times 6} = 160$  and  $L_{Sm_{8 \times 8}} = 200$ .

**Example 4.13.** Let's consider a following magic square with *even* number magic sum based on the Result 4.9:

3b	mgc	S=3*T/4							200
	12	814	15	-586	14	13	11	-93	200
	9	78	14	17	21	25	-5	41	200
	8	40	15	18	22	26	29	42	200
	7	-4	55	19	23	27	30	43	200
	6	11	-21	77	24	28	31	44	200
	5	12	16	-1	110	-19	32	45	200
	10	13	71	20	-50	63	33	40	200
	143	-764	35	636	36	37	39	38	200
	200	200	200	200	200	200	200	200	200

The **magic** sums of above magic squares are  $S_{6 \times 6} = 150$  and  $T_{8 \times 8} = 200$ . It is a magic square as it satisfies the condition given in (2).

## 5 Magic Squares of Order 10

Below are three examples of magic squares of order 10.

**Example 5.1.** Let's consider a magic square of order 10



mgc	10x10 <a href="https://numbers-magic.com/">https://numbers-magic.com/</a>										505
	69	87	8	27	10	99	72	95	25	13	505
	14	32	93	74	91	2	29	6	88	76	505
	85	16	49	60	61	38	33	62	24	77	505
	84	17	68	59	47	35	52	42	18	83	505
	90	11	65	41	34	63	64	36	3	98	505
	23	78	39	53	43	54	48	66	96	5	505
	12	89	45	46	67	55	50	40	71	30	505
	79	22	37	44	51	58	56	57	4	97	505
	21	73	75	86	70	1	9	81	82	7	505
	28	80	26	15	31	100	92	20	94	19	505
	505	505	505	505	505	505	505	505	505	505	505

It is a **double digits bordered** magic squares where the magic rectangles of order  $2 \times 4$  are of equal sums.

**Example 5.2.** Let's consider a magic square of order 10

mgc	10x10 <a href="https://numbers-magic.com/">https://numbers-magic.com/</a>										505
	49	54	43	56	61	40	31	70	18	83	505
	44	55	50	53	41	60	19	82	5	96	505
	58	45	52	47	59	42	72	29	94	7	505
	51	48	57	46	67	34	27	74	99	2	505
	36	35	62	68	38	64	25	76	95	6	505
	65	66	39	33	37	63	71	30	10	91	505
	28	26	80	69	24	79	78	20	88	13	505
	73	75	21	32	77	22	81	23	8	93	505
	15	84	12	90	9	98	97	85	1	14	505
	86	17	89	11	92	3	4	16	87	100	505
	505	505	505	505	505	505	505	505	505	505	505

It is a **cornered** magic squares, where in each case the magic rectangles of orders  $2 \times 4$ ,  $2 \times 6$ ,  $2 \times 8$  are of equal sums. It contains magic squares of orders 4, 6 and 8. In the left upper corner there is a magic square of order 4.

**Example 5.3.** Let's consider a magic square of order 10

mgc	10x10 <a href="https://numbers-magic.com/">https://numbers-magic.com/</a>									505
91	86	16	84	18	14	4	98	2	92	505
13	26	20	80	82	69	31	71	25	88	505
89	23	64	62	35	68	36	38	78	12	505
11	24	34	49	54	43	56	67	77	90	505
96	29	40	44	55	50	53	61	72	5	505
1	79	42	58	45	52	47	59	22	100	505
93	74	60	51	48	57	46	41	27	8	505
7	73	63	39	66	33	65	37	28	94	505
95	76	81	21	19	32	70	30	75	6	505
9	15	85	17	83	87	97	3	99	10	505
505	505	505	505	505	505	505	505	505	505	505

It is a well known *single digits bordered* magic square. It also contains magic squares of orders 4, 6 and 8.

For more details on these kind of magic squares refer author’s work [25, 26, 27, 28].

### 5.1 Reduced Entries Magic Squares of Order 10

Below are 6 results for reduced entries magic squares of order 10 based on the Examples 5.1, 5.2 and 5.3.

**Result 5.1.** *Let’s consider following magic square of order 10 with **reduced entries**:*

$2*(T-S)/2-A35-A20-A41$	$2*A41+A35+A34+A20-A21-2*A28-3*T/2+5*S/2-A27$	A15	A16	A17	A18	$3*(T-S)/2-A15-A16-A17-A18-A19$	A19	$A21+2*A28-A20+A27-A41-A34$	A20
A27	A28	$(T-S)/2-A15$	$(T-S)/2-A16$	$(T-S)/2-A17$	$(T-S)/2-A18$	$A15+A16+A17+A18+A19-2*(T-S)/2$	$(T-S)/2-A19$	$3*S/2-A21-T/2-A27-A28$	A21
$3*(T-S)/2-A22-A23-A24-A25-A26$	$A22+A23+A24+A25+A26-2*(T-S)/2$	A1	$A4+M-A6-A1$	$A6-A3-A4$	A3	S-M-A10	A10	$(T-S)/2-A29$	A29
A22	$(T-S)/2-A22$	A6	A4	A5	$M-A4-A5-A6$	$A10+A11+A12-S+M$	$2*S-2*M-A10-A11-A12$	$(T-S)/2-A30$	A30
A23	$(T-S)/2-A23$	$A2+A3-A6$	$A1+A2-A5$	$M-A1-A2-A4$	$A4+A5+A6-A2-A3$	S-M-A11	A11	$A29+A30+A31+A32+A33-T+S$	$3*(T-S)/2-A29-A30-A31-A32-A33$
A24	$(T-S)/2-A24$	$M-A1-A2-A3$	$A5-A2-2*A4+A6$	$A1+A2+A3+2*A4-A5-A6$	A2	S-M-A12	A12	$(T-S)/2-A31$	A31
A25	$(T-S)/2-A25$	S-M-A8	$2*S-A8-A1-3*A4-2*A10-A11-A12-M$	$2*A8+A1+A9+3*A4+2*A10+A11+A12+M-2*S$	S-M-A9	$(1/2)*(S-M)$	$(5*M-3*S)/2$	$(T-S)/2-A32$	A32
A26	$(T-S)/2-A26$	A8	$A8+A1+3*A4+2*A10+A11+A12-S$	$3*S-2*A8-A1-A9-3*A4-2*A10-A11-A12-2*M$	A9	$(5*M-3*S)*(1/2)$	$(S-M)/2$	$(T-S)/2-A33$	A33
$5*S/2-3*T/2+A35+A20-A27$	$3*T/2-5*S/2-A20+A27+A28-A41+A21$	$(T-S)/2-A36$	$A36+A37+A38+A39+A40-T+S$	$(T-S)/2-A37$	$(T-S)/2-A38$	$(T-S)/2-A39$	$(T-S)/2-A40$	$A41+A20-A28$	$3*S/2-A35-T/2-A20-A21$
A41	$3*S/2-T/2-A41-A35-A34$	A36	$3*(T-S)/2-A36-A37-A38-A39-A40$	A37	A38	A39	A40	A34	A35

### Details:

It is a **double-digit bordered** magic square of order 10 embedded with **cornered** magic square of order 6 having magic square of order 4 at the left upper corner. The two magic rectangles of orders  $2 \times 4$  are of equal width and length. Also the four magic rectangles of orders  $2 \times 6$  are of equal width and length. The letters M, S and L represents the magic squares of orders 4, 8 and 10 respectively. In order to avoid decimal entries, the magic sums M, S and L should be of same type, i.e., either **even** or **odd** numbers.

**Example 5.4.** Let's consider following two examples:

1a	mgc	10x10 <a href="https://numbers-magic.com/">https://numbers-magic.com/</a> ©IJT										200
	-76	152	25	26	27	28	-60	29	19	30		200
	37	38	0	-1	-2	-3	85	-4	19	31		200
	-95	120	11	77	-11	13	40	20	-14	39		200
	32	-7	16	14	15	45	3	57	-15	40		200
	33	-8	9	8	53	20	39	21	155	-130		200
	34	-9	54	-9	33	12	38	22	-16	41		200
	35	-10	42	56	-19	41	30	0	-17	42		200
	36	-11	18	4	79	19	0	30	-18	43		200
	113	-50	-21	190	-22	-23	-24	-25	43	19		200
	51	-15	46	-165	47	48	49	50	44	45		200
	200	200	200	200	200	200	200	200	200	200		200

1a	mgc	10x10 <a href="https://inderjtaneja.wordpress.com/">https://inderjtaneja.wordpress.com/</a> ©IJT										221
	-56	123	25	26	27	28	-30	29	19	30		221
	37	38	10	9	8	7	65	6	10	31		221
	-65	100	11	78	-11	13	40	20	-4	39		221
	32	3	16	14	15	46	3	57	-5	40		221
	33	2	9	8	54	20	39	21	135	-100		221
	34	1	55	-9	33	12	38	22	-6	41		221
	35	0	42	57	-20	41	30	1	-7	42		221
	36	-1	18	3	80	19	1	30	-8	43		221
	84	-21	-11	170	-12	-13	-14	-15	43	10		221
	51	-24	46	-135	47	48	49	50	44	45		221
	221	221	221	221	221	221	221	221	221	221		221

The magic sums of above two examples are as follows:

**First Example**

$$M_{4 \times 4} = 90$$

$$S_{8 \times 8} = 150$$

$$L_{12 \times 12} = 200$$

**Second Example**

$$M_{4 \times 4} = 91$$

$$S_{8 \times 8} = 151$$

$$L_{12 \times 12} = 221$$

Above two examples are considered with magic sums as **even** and **odd** numbers. The magic rectangles sums are:

**First Example**

$$MR_{2 \times 4} = 60 \times 120$$

$$MR_{2 \times 6} = 25 \times 75$$

**Second Example**

$$MR_{2 \times 4} = 60 \times 120$$

$$MR_{2 \times 6} = 35 \times 105$$

**Result 5.2.** Let's consider following magic square of order 10 with *reduced entries*:

A18	A19	$2*(T-S)/2-A15-A16-A17$	A15	A16	A17	$2*S-T+A18-A19+A23$	$2*(T-S)/2-A23-2*A18$	L-T-A28	A28
$2*A13+A24+A14+2*A18-2*(T-S)/2-A19$	$2*(T-S)/2-2*A18-A13$	$A15+A16+A17-(T-S)/2$	$(T-S)/2-A15$	$(T-S)/2-A16$	$(T-S)/2-A17$	$2*(T-S)/2-A23-A13-A24-A18+A19$	$2*S-T+A18+A23-A14$	L-T-A29	A29
$2*(T-S)/2-A7-A8-A9$	$A7+A8+A9-(T-S)/2$	A1	$A4+S-A6-A1$	$A6-A3-A4$	A3	$A10+A11+A12-(T-S)/2$	$2*(T-S)/2-A10-A11-A12$	$A28+A29+A30+A31+A32+A33+A34+3*T-3*L$	$4*L-A28-A29-A30-A31-A32-A33-A34-4*T$
A7	$(T-S)/2-A7$	A6	A4	A5	$S-A4-A5-A6$	$(T-S)/2-A10$	A10	L-T-A30	A30
A8	$(T-S)/2-A8$	$A2+A3-A6$	$A1+A2-A5$	$S-A1-A2-A4$	$A4+A5+A6-A2-A3$	$(T-S)/2-A11$	A11	L-T-A31	A31
A9	$(T-S)/2-A9$	$S-A1-A2-A3$	$A5-A2-2*A4+A6$	$A1+A2+A3+2*A4-A5-A6$	A2	$(T-S)/2-A12$	A12	L-T-A32	A32
$T-2*A13-A14+A19-3*A18-A23-A24$	$A23+3*A18+A24-A19+A13-2*(T-S)/2$	$A20+A21+A22-(T-S)/2$	$(T-S)/2-A20$	$(T-S)/2-A21$	$(T-S)/2-A22$	A13	A14	L-T-A33	A33
A23	$T-A23-A18-2*(T-S)/2-A24$	$2*(T-S)/2-A20-A21-A22$	A20	A21	A22	A24	A18	L-T-A34	A34
L-T-A35	$T-A1+A29-A28+A11+A21+A22+2*A20-6*(T-S)/2-A35+2*A10+A12-3*A4$	$T+A1-A29+A28-A11-A21-A22-2*A20+6*(T-S)/2+2*A35-2*A10-A12-2*L+3*A4+A36+A37+A38+A39+A40$	L-T-A36	L-T-A37	L-T-A38	L-T-A39	L-T-A40	$(L-T)/2$	$(9*T-7*L)/2$
A35	$A1-A29+A28-A11-A21-A22-2*A20+6*(T-S)/2+A35-2*T-2*A10-A12+L+3*A4$	$3*L-2*T-A1+A29-A28+A11+A21+A22+2*A20-6*(T-S)/2-2*A35+2*A10+A12-3*A4-A36-A37-A38-A39-A40$	A36	A37	A38	A39	A40	$(9*T-7*L)/2$	$(L-T)/2$

### Details:

It is a **cornered** magic square of order 10 with **double-digit bordered** magic square of order 8 with magic square of order 4 in the middle. The four magic rectangles of orders  $2 \times 4$  are of equal width and length. Also the two magic rectangles of orders  $2 \times 8$  are of equal width and length. The letters M, T and L represents the magic squares of orders 4, 8 and 10 respectively. In order to avoid decimal entries, the magic sums M, T and L should be of same type, i.e., either **even** or **odd**.

**Example 5.5.** Let's consider following two examples:



2a	mgc	10x10 <a href="https://numbers-magic.com/">https://numbers-magic.com/</a> ©IJT										220
	45	47	-63	39	41	43	103	-85	-15	65		220
	147	-65	93	-9	-11	-13	-85	113	-17	67		220
	-15	45	11	95	-11	15	63	-33	347	-297		220
	23	7	21	17	19	53	1	29	-19	69		220
	25	5	7	5	69	29	-1	31	-21	71		220
	27	3	71	-7	33	13	-3	33	-23	73		220
	-137	175	123	-19	-21	-23	35	37	-25	75		220
	55	-47	-93	49	51	53	57	45	-27	77		220
	-29	175	229	-31	-33	-35	-37	-39	25	-5		220
	79	-125	-179	81	83	85	87	89	-5	25		220
	220	220	220	220	220	220	220	220	220	220		220

2b	mgc	10x10 <a href="https://inderjtaneja.wordpress.com/">https://inderjtaneja.wordpress.com/</a> ©IJT										251
	45	47	-63	39	41	43	118	-85	1	65		251
	147	-65	93	-9	-11	-13	-85	128	-1	67		251
	-15	45	11	110	-11	15	63	-33	299	-233		251
	23	7	21	17	19	68	1	29	-3	69		251
	25	5	7	5	84	29	-1	31	-5	71		251
	27	3	86	-7	33	13	-3	33	-7	73		251
	-122	175	123	-19	-21	-23	35	37	-9	75		251
	55	-32	-93	49	51	53	57	45	-11	77		251
	-13	190	182	-15	-17	-19	-21	-23	33	-46		251
	79	-124	-116	81	83	85	87	89	-46	33		251
	251	251	251	251	251	251	251	251	251	251		251

The magic sums of above two examples are as follows:

**First Example**

$$M_{4 \times 4} = 100$$

$$T_{8 \times 8} = 170$$

$$L_{10 \times 10} = 220$$

**Second Example**

$$M_{4 \times 4} = 125$$

$$T_{8 \times 8} = 185$$

$$L_{10 \times 10} = 251$$

Above two examples are considered with magic sums as **even** and **odd** numbers. The magic rectangles sums are:

**First Example**

$$MR_{2 \times 4} = 30 \times 60$$

$$MR_{2 \times 8} = 50 \times 200$$

**Second Example**

$$MR_{2 \times 4} = 30 \times 60$$

$$MR_{2 \times 8} = 66 \times 264$$

**Result 5.3.** Let's consider following magic square of order 10 with *reduced entries*:



A1	A4+M-A6-A1	A6-A3-A4	A3	S-M-A10	A10	a18	T-S-A18	L-T-A30	A30
A6	A4	A5	M-A4-A5-A6	S-M-A11	A11	a19	T-S-A19	L-T-A31	A31
A2+A3-A6	A1+A2-A5	M-A1-A2-A4	A4+A5+A6-A2-A3	M-S+A10+A11+A12	2*(S-M)-A10-A11-A12	A20	T-S-A20	L-T-A32	A32
M-A1-A2-A3	A5-A2-2*A4+A6	A1+A2+A3+2*A4-A5-A6	A2	S-M-A12	A12	3*(T-S)-A18-A19-A20-A21-A22	2*(S-T)+A18+A19+A20+A21+A22	L-T-A33	A33
2*S-3*M+A1-A8+3*A4-A11+A10	S-M-A8	3*M-2*S-A1+2*A8-3*A4+A11-A10+A9	S-M-A9	(S-M)/2	(5*M-3*S)/2	A21	T-S-A21	3*(T-L)+A30+A31+A32+A33+A34+A35+A36	4*(L-T)-A30-A31-A32-A33-A34-A35-A36
2*M-S-A1+A8-3*A4+A11-A10	A8	3*S-4*M+A1-2*A8+3*A4-A11+A10-A9	A9	(5*M-3*S)/2	(S-M)/2	A22	T-S-A22	L-T-A34	A34
A14	2*A11-T-5*S-A1+2*A9+2*A8+2*A12+A14-3*A4+A18-A19+8*M	A15	4*T+2*S-2*A14+A1-2*A11-2*A9-2*A8-2*A12+3*A4-A18+A19-8*M-A15-A16-A17	A16	A17	(T-S)/2	(7*S-5*T)/2	L-T-A35	A35
T-S-A14	2*T+4*S+A1-2*A11-2*A9-2*A8-2*A12-A14+3*A4-A18+A19-8*M	T-S-A15	2*A14-3*S-3*T-A1+2*A11+2*A9+2*A8+2*A12-3*A4+A18-A19+8*M+A15+A16+A17	T-S-A16	T-S-A17	(7*S-5*T)/2	(T-S)/2	L-T-A36	A36
2*L+6*T-6*S-2*A20-A16-2*A15-2*A11-2*A9-A17-2*A8-A22-2*A12-A21-2*A14+3*A4-2*A18+A1-A24+A30-A31-3*M	L-T-A24	L-T-A25	L-T-A26	L-T-A27	6*S-4*L-4*T+2*A20+A29+A27+A16+2*A15+2*A11+2*A9+A17+2*A8+A25+A28+A22+A26-A1+2*A12+A21+2*A14-3*A4+2*A18+2*A24-A30+A31+3*M	L-T-A28	L-T-A29	(L-T)/2	(9*T-7*L)/2
6*S-L-7*T+2*A20+A16+2*A15+2*A11+2*A9+A17+2*A8+A22+2*A12+A21+2*A14-3*A4+2*A18-A1+A24-A30+A31+3*M	A24	A25	A26	A27	5*L+3*T-6*S-2*A20-A29-A27-A16-2*A15-2*A11-2*A9-A17-2*A8-A25-A28-A22-A26+A1-2*A12-A21-2*A14+3*A4-2*A18-2*A24+A30-A31-3*M	A28	A29	(9*T-7*L)/2	(L-T)/2

### Details:

It is a **cornered** magic square of order 10 having the magic squares of orders 6 and 8 also **cornered** magic squares. The two magic rectangles of orders  $2 \times 4$ ,  $2 \times 6$  and  $2 \times 8$  are of equal width and length in each case. The letters M, S, T and L represents the magic squares of orders 4, 6, 8 and 10 respectively. In order to avoid decimal entries, the magic sums M, S, T and L should be of same type, i.e., either **even** or **odd** numbers.

**Example 5.6.** Let's consider following two examples:

3a	mgc	10x10 <a href="https://numbers-magic.com/">https://numbers-magic.com/</a> ©IJT										222
	21	67	-21	23	0	30	25	37	-3	43		222
	26	24	25	15	63	-33	52	10	-4	44		222
	19	18	23	30	-1	31	40	22	202	-162		222
	24	-19	63	22	-2	32	24	38	-5	45		222
	2	-94	151	1	15	45	23	39	-6	46		222
	28	124	-121	29	45	15	22	40	-7	47		222
	38	160	-93	28	27	26	31	-35	-8	48		222
	24	-98	155	34	35	36	-35	31	-9	49		222
	166	-10	69	-11	-12	-13	-14	-15	20	42		222
	-126	50	-29	51	52	53	54	55	42	20		222
	222	222	222	222	222	222	222	222	222	222		222
3b	mgc	10x10 <a href="https://inderjtaneja.wordpress.com/">https://inderjtaneja.wordpress.com/</a> ©IJT										301
	21	68	-21	23	10	30	23	37	67	43		301
	26	24	25	16	53	-13	19	41	66	44		301
	19	18	24	30	9	31	75	-15	-8	118		301
	25	-19	63	22	8	32	22	38	65	45		301
	12	-73	130	11	20	31	21	39	64	46		301
	28	113	-90	29	31	20	20	40	63	47		301
	66	181	-142	26	25	24	30	-19	62	48		301
	-6	-121	202	34	35	36	-19	30	61	49		301
	275	60	-180	59	58	57	56	55	55	-194		301
	-165	50	290	51	52	53	54	55	-194	55		301
	301	301	301	301	301	301	301	301	301	301		301

The magic sums of above two examples are as follows:

#### First Example

$$M_{4 \times 4} = 90$$

$$S_{6 \times 6} = 120$$

$$T_{8 \times 8} = 182$$

$$L_{10 \times 10} = 222$$

#### Second Example

$$M_{4 \times 4} = 91$$

$$S_{6 \times 6} = 131$$

$$T_{8 \times 8} = 191$$

$$L_{10 \times 10} = 301$$

Above two examples are considered with magic sums as **even** and **odd** numbers. The magic rectangles sums are:

#### First Example

$$MR_{2 \times 4} = 30 \times 60$$

$$MR_{2 \times 6} = 62 \times 186$$

$$MR_{2 \times 8} = 40 \times 160$$

#### Second Example

$$MR_{2 \times 4} = 30 \times 60$$

$$MR_{8 \times 6} = 60 \times 180$$

$$MR_{2 \times 8} = 110 \times 440$$

**Result 5.4.** Let's consider following magic square of order 10 with *reduced entries*:

$4*S-6*M+A12$	$S+2*A13+2*A14-4*A12-M$	$2*A12+9*M-6*S-A13-A14$	$S-A13-M$	$S-A14-M$	$A12$	$T-S-A16$	$A16$	$L-T-A25$	$A25$
$4*M-3*S+A9+A10+A11$	$A1$	$A4+M-A6-A1$	$A6-A3-A4$	$A3$	$4*S-A9-A10-A11-5*M$	$T-S-A16-A1-A13-A9-A14+2*A12-7*S-3*A4+10*M$	$A16+A1+A13+A9+A14-2*A12+7*S+3*A4-10*M$	$3*T-3*L+A25+A26+A27+A28+A29+A30+A31$	$4*L-4*T-A25-A26-A27-A28-A29-A30-A31$
$S-A9-M$	$A6$	$A4$	$A5$	$M-A4-A5-A6$	$A9$	$2*A16+A18+A19+A1-2*T+9*S+A13+A9+A14-2*A12+3*A4-10*M+A17$	$3*T-10*S-A18-A19-2*A16-A1-A13-A9-A14+2*A12-3*A4+10*M-A17$	$L-T-A26$	$A26$
$S-A10-M$	$A2+A3-A6$	$A1+A2-A5$	$M-A1-A2-A4$	$A4+A5+A6-A2-A3$	$A10$	$T-S-A17$	$A17$	$L-T-A27$	$A27$
$S-A11-M$	$M-A1-A2-A3$	$A5-A2-2*A4+A6$	$A1+A2+A3+2*A4-A5-A6$	$A2$	$A11$	$T-S-A18$	$A18$	$L-T-A28$	$A28$
$5*M-A12-3*S$	$4*A12-2*A13-2*A14$	$7*S+A13+A14-2*A12-10*M$	$A13$	$A14$	$5*M-A12-3*S$	$T-S-A19$	$A19$	$L-T-A29$	$A29$
$T-S+A13+A9+A14-2*A12+2*A1+7*S+6*A4-11*M$	$8*S+A13+A9+A14-2*A12+2*A1+6*A4-11*M$	$A21+A20+A22-T-2*A13-2*A9-2*A14+4*A12-4*A1-14*S-12*A4+22*M$	$T-S-A20$	$T-S-A21$	$T-S-A22$	$(T-S)/2$	$(7*S-5*T)/2$	$L-T-A30$	$A30$
$11*M-A13-A9-A14+2*A12-2*A1-7*S-6*A4$	$T-9*S-A13-A9-A14+2*A12-2*A1-6*A4+11*M$	$2*T+13*S+2*A13+2*A9+2*A14-4*A12+4*A1+12*A4-22*M-A21-A20-A22$	$A20$	$A21$	$A22$	$(7*S-5*T)/2$	$(T-S)/2$	$L-T-A31$	$A31$
$L-T-A32$	$4*L+2*A17+A19+A18+2*A16+12*M-9*A4+2*A12-3*A1-S-A13-A9-2*A14-A11+A21+2*A20+A22-A32-10*T-2*A25-A26-A27-A28-A29-A30-A31$	$2*A32+12*T+2*A25+A26+A27+A28+A29+A30+A31-6*L+A37+A36+A35+A34+A33-2*A17-A19-A18-2*A16-12*M+9*A4-2*A12+3*A1+S+A13+A9+2*A14+A11-A21-2*A20-A22$	$L-T-A33$	$L-T-A34$	$L-T-A35$	$L-T-A36$	$L-T-A37$	$(L-T)/2$	$(9*T-7*L)/2$
$A32$	$A32+9*T+2*A25+A26+A27+A28+A29+A30+A31-3*L-2*A17-A19-A18-2*A16-12*M+9*A4-2*A12+3*A1+S+A13+A9+2*A14+A11-A21-2*A20-A22$	$2*A17+A19+A18+2*A16+12*M-9*A4+2*A12-3*A1-S-A13-A9-2*A14-A11+A21+2*A20+A22-2*A32-13*T-2*A25-A26-A27-A28-A29-A30-A31+7*L-A37-A36-A35-A34-A33$	$A33$	$A34$	$A35$	$A36$	$A37$	$(9*T-7*L)/2$	$(L-T)/2$

### Details:

It is also a **cornered** magic square of order 10 with **single-digit bordered** magic square of order 6 at the upper left corner containing magic square of order 4. The magic rectangles of orders  $2 \times 6$  and  $2 \times 8$  are of equal width and length in each case. The letters M, S, T and L represents the magic squares of orders 4, 6, 8 and 10 respectively. In order to avoid decimal entries, the magic sums M, S, T and L should be of same type, i.e., either **even** or **odd** numbers.

**Example 5.7.** Let's consider following two examples:

4a	mgc	10x10 <a href="https://numbers-magic.com/">https://numbers-magic.com/</a> ©IJT										250
	84	68	-69	3	0	44	4	56	-23	83		250
	44	11	63	-11	17	6	-221	281	464	-404		250
	15	26	20	23	11	35	403	-343	-26	86		250
	12	5	2	35	38	38	1	59	-29	89		250
	9	38	-5	33	14	41	-2	62	-32	92		250
	-34	-18	119	47	50	-34	-5	65	-35	95		250
	276	346	-409	-8	-11	-14	30	-20	-38	98		250
	-216	-286	469	68	71	74	-20	30	-41	101		250
	-44	-611	1160	-47	-50	-53	-56	-59	30	-20		250
	104	671	-1100	107	110	113	116	119	-20	30		250
	250	250	250	250	250	250	250	250	250	250		250

4b	mgc	10x10 <a href="https://inderjtaneja.wordpress.com/">https://inderjtaneja.wordpress.com/</a> ©IJT										333
	128	99	-135	34	31	44	-6	56	-1	83		333
	-9	11	103	-11	17	90	-328	378	398	-316		333
	46	26	20	23	51	35	520	-470	-4	86		333
	43	5	2	75	38	38	-9	59	-7	89		333
	40	78	-5	33	14	41	-12	62	-10	92		333
	-47	-18	216	47	50	-47	-15	65	-13	95		333
	323	474	-584	-18	-21	-24	25	76	-16	98		333
	-273	-424	634	68	71	74	76	25	-19	101		333
	-22	-480	985	-25	-28	-31	-34	-37	41	-36		333
	104	562	-903	107	110	113	116	119	-36	41		333
	333	333	333	333	333	333	333	333	333	333		333

The magic sums of above two examples are as follows:

#### First Example

$$M_{4 \times 4} = 80$$

$$S_{6 \times 6} = 130$$

$$T_{8 \times 8} = 190$$

$$L_{10 \times 10} = 250$$

#### Second Example

$$M_{4 \times 4} = 120$$

$$S_{6 \times 6} = 201$$

$$T_{8 \times 8} = 251$$

$$L_{10 \times 10} = 333$$

Above two examples are considered with magic sums as **even** and **odd** numbers. The magic rectangles sums are:

#### First Example

$$MR_{2 \times 6} = 60 \times 180$$

$$MR_{2 \times 8} = 60 \times 240$$

#### Second Example

$$MR_{2 \times 6} = 50 \times 150$$

$$MR_{2 \times 8} = 82 \times 328$$

**Result 5.5.** Let's consider following magic square of order 10 with *reduced entries*:

T-S-A17	$2*T+3*S+A19-A14-A20$	T-S-A14	$A18+2*A17-7*T+2*S+A16+A15+2*A14+A21+A22+A24+2*A20+A23$	T-S-A15	T-S-A16	T-S-A18	T-A21-A22-A24-A20-A23-A17-A19	A32	L-T-A32
T-S-A20	A1	A4+M-A6-A1	A6-A3-A4	A3	S-M-A10	A10	A20	A33	L-T-A33
T-S-A21	A6	A4	A5	M-A4-A5-A6	$A10+A11+A12-S+M$	$2*S-2*M-A10-A11-A12$	A21	A34	L-T-A34
T-S-A22	A2+A3-A6	A1+A2-A5	M-A1-A2-A4	A4+A5+A6-A2-A3	S-M-A11	A11	A22	A35	L-T-A35
T-S-A23	M-A1-A2-A3	A5-A2-2*A4+A6	$A1+A2+A3+2*A4-A5-A6$	A2	S-M-A12	A12	A23	$4*L-4*T-A32-A33-A34-A35-A36-A37-A38$	$3*(T-L)+A32+A33+A34+A35+A36+A37+A38$
T-S-A24	S-M-A8	$2*S-A8-A1-3*A4-2*A10-A11-A12-M$	$2*A8+A1+A9+3*A4+2*A10+A11+A12+M-2*S$	S-M-A9	(S-M)/2	$(5*M-3*S)/2$	A24	A36	L-T-A36
T-S-A19	A8	$A8+A1+3*A4+2*A10+A11+A12-S$	$3*S-2*A8-A1-A9-3*A4-2*A10-A11-A12-2*M$	A9	$(5*M-3*S)/2$	(S-M)/2	A19	A37	L-T-A37
$A17+A19+7*S+A21+A22+A24+A20+A23-6*T$	$A14+A20-T-A19-4*S$	A14	$8*T-A18-2*A17-3*S-A16-A15-2*A14-A21-A22-A24-2*A20-A23$	A15	A16	A18	A17	A38	L-T-A38
$5*S-2*A12-4*M+A21-3*A4-2*A10+A33-2*T-A1+A26-2*A8-2*A9-A32+L+A14$	A26	A27	A28	$3*L-2*T+2*A12-5*S+4*M-A21+3*A4+2*A10-A33+A1-2*A26+2*A8+2*A9+A32-A27-A28-A29-A30-A31-A14$	A29	A30	A31	(L-T)/2	$(9*T-7*L)/2$
$T+2*A12-5*S+4*M-A21+3*A4+2*A10-A33+A1-A26+2*A8+2*A9+A32-A14$	L-T-A26	L-T-A27	L-T-A28	$T-2*L-2*A12+5*S-4*M+A21-3*A4-2*A10+A33-A1+2*A26-2*A8-2*A9-A32+A27+A28+A29+A30+A31+A14$	L-T-A29	L-T-A30	L-T-A31	$(9*T-7*L)/2$	(L-T)/2

### Details:

It is also a **cornered** magic square of order 10 with **single-digit bordered** magic square of order 8 at the upper left corner. The inner part is again a **cornered** magic square of order 6 embedded with magic square of order 4. The magic rectangles of orders  $2 \times 4$  and  $2 \times 8$  are of equal width and length in each case. The letters M, S, T and L represents the magic squares of orders 4, 6, 8 and 10 respectively. In order to avoid decimal entries, the magic sums pairs (M, S) and (T, L) should be of same type, i.e., either **even** or **odd** numbers.

**Example 5.8.** Let's consider following two examples:



5a	mgc	10x10 <a href="https://numbers-magic.com/">https://numbers-magic.com/</a> ©IJT										236
	-3	701	3	-271	1	-1	-5	-245	-27	83		236
	-9	21	85	-21	25	-19	39	59	-29	85		236
	-11	31	27	29	23	103	-83	61	455	-399		236
	-13	17	15	39	39	-21	41	63	-31	87		236
	-15	41	-17	63	23	-23	43	65	-33	89		236
	-17	-15	17	55	-17	10	80	67	-35	91		236
	-7	35	3	-35	37	80	10	57	-37	93		236
	255	-651	47	321	49	51	55	53	-39	95		236
	-67	-15	411	-17	-19	-21	-23	-25	28	-16		236
	123	71	-355	73	75	77	79	81	-16	28		236
	236	236	236	236	236	236	236	236	236	236		236

5b	mgc	10x10 <a href="https://inderjtaneja.wordpress.com/">https://inderjtaneja.wordpress.com/</a> ©IJT										285
	8	723	14	-348	12	10	6	-234	11	83		285
	2	21	85	-21	25	-19	39	59	9	85		285
	0	31	27	29	23	103	-83	61	341	-247		285
	-2	17	15	39	39	-21	41	63	7	87		285
	-4	41	-17	63	23	-23	43	65	5	89		285
	-6	-15	17	55	-17	10	80	67	3	91		285
	4	35	3	-35	37	80	10	57	1	93		285
	189	-662	47	409	49	51	55	53	-1	95		285
	-2	23	270	21	19	17	15	13	47	-138		285
	96	71	-176	73	75	77	79	81	-138	47		285
	285	285	285	285	285	285	285	285	285	285		285

The magic sums of above two examples are as follows:

**First Example**

$$M_{4 \times 4} = 110$$

$$S_{6 \times 6} = 130$$

$$T_{8 \times 8} = 180$$

$$L_{10 \times 10} = 236$$

**Second Example**

$$M_{4 \times 4} = 110$$

$$S_{6 \times 6} = 130$$

$$L_{10 \times 10} = 191$$

$$R_{12 \times 12} = 285$$

Above two examples are considered with magic sums as **even** and **odd** numbers. The magic rectangles sums are:

**First Example**

$$MR_{2 \times 4} = 20 \times 40$$

$$MR_{2 \times 8} = 56 \times 224$$

**Second Example**

$$MR_{2 \times 4} = 20 \times 40$$

$$MR_{2 \times 8} = 94 \times 376$$

**Result 5.6.** Let's consider following magic square of order 10 with *reduced entries*:



	$2*T+3*S+A16-2*A13-A22-2*A12-2*A8-5*M-3*A4-A17-2*A14$	T-S-A22	$A26+2*A25+2*A13-7*T+2*S+A24+A23+2*A22+2*A12+2*A8+5*M+3*A4+A18+A19+A21+2*A17+2*A14+A20$	T-S-A23	T-S-A24	T-S-A26	T-A18-A19-A21-A17-A20-A25-A16	L-T-A29	A29
T-S-A17	S-M-A14	S-A12-M	$2*A11-3*A4-A12-A8+A9+M$	$3*A4-3*S+2*A12+A13+2*A8+A9-A10+2*A14+2*M$	S-A13-M	S-A8-A9-A10-A11-A14	A17	L-T-A30	A30
T-S-A18	S-M-A9	A1	A4+M-A6-A1	A6-A3-A4	A3	A9	A18	$A29+A30+A31+A32+A33+A34+A35+3*T-3*L$	$4*L-A29-A30-A31-A32-A33-A34-A35-4*T$
T-S-A19	S-M-A10	A6	A4	A5	M-A4-A5-A6	A10	A19	L-T-A31	A31
T-S-A20	S-M-A11	A2+A3-A6	A1+A2-A5	M-A1-A2-A4	A4+A5+A6-A2-A3	A11	A20	L-T-A32	A32
T-S-A21	S-M-A8	M-A1-A2-A3	A5-A2-2*A4+A6	$A1+A2+A3+2*A4-A5-A6$	A2	A8	A21	L-T-A33	A33
T-S-A16	$A8-4*S+A9+A10+A11+A14+5*M$	A12	$S-2*A11+3*A4+A12+A8-A9-2*M$	$4*S+A11-3*A4-2*A12-A13-2*A8-A10-2*A14-3*M$	A13	A14	A16	L-T-A34	A34
$A25+A16+7*S+A18+A19+A21+A17+A20-6*T$	$A22+2*A12+2*A8+5*M+3*A4+A17+2*A14-T-A16+2*A13-4*S$	A22	$8*T-A26-2*A25-2*A13-3*S-A24-A23-2*A22-2*A12-2*A8-5*M-3*A4-A18-A19-A21-2*A17-2*A14-A20$	A23	A24	A26	A25	L-T-A35	A35
L-T-A36	$A30-A29-A18-A36+T-A12-A22+2*A11+A9-S-A8-6*A4+2*M-A1-A10$	$A29-A30+A18+A38+A37+A40+2*A36+T+A12+A22-2*A11+A39-A9-2*L+S+A8+6*A4-2*M+A1+A10+A41$	L-T-A37	L-T-A38	L-T-A39	L-T-A40	L-T-A41	(L-T)/2	$(9*T-7*L)/2$
A36	$A29-A30+A18+A36-2*T+A12+A22-2*A11-A9+L+S+A8+6*A4-2*M+A1+A10$	$A30-A29-A18-A38-A37-A40-2*A36-2*T-A12-A22+2*A11-A39+A9+3*L-S-A8-6*A4+2*M-A1-A10-A41$	A37	A38	A39	A40	A41	$(9*T-7*L)/2$	(L-T)/2

### Details:

It is also a **cornered** magic square of order 10 with **single-digit bordered** magic square of order 8 at the upper left corner. The inner part is magic square of order 4. The magic rectangles of order  $2 \times 8$  are of equal width and length in each case. The letters M, S, T and L represents the magic squares of orders 4, 6, 8 and 10 respectively. In order to avoid decimal entries, the magic sums pairs (T, L) should be of same type, i.e., either **even** or **odd** numbers.

**Example 5.9.** Let's consider following two examples:

6a	mgc	10x10 <a href="https://numbers-magic.com/">https://numbers-magic.com/</a> ©IJT										320
	35	291	38	-265	37	36	34	64	11	39		320
	43	56	58	99	-168	57	98	27	10	40		320
	42	61	11	107	-11	13	19	28	144	-94		320
	41	60	16	14	15	75	20	29	9	41		320
	40	59	9	8	83	20	21	30	8	42		320
	39	62	84	-9	33	12	18	31	7	43		320
	44	-98	22	-19	248	23	24	26	6	44		320
	-14	-221	32	335	33	34	36	35	5	45		320
	4	111	80	3	2	1	0	-1	25	95		320
	46	-61	-30	47	48	49	50	51	95	25		320
	320	320	320	320	320	320	320	320	320	320		320

6b	mgc	10x10 <a href="https://inderjtaneja.wordpress.com/">https://inderjtaneja.wordpress.com/</a> ©IJT										351
	36	293	39	-272	38	37	35	65	41	39		351
	44	56	58	99	-168	57	98	27	40	40		351
	43	61	11	107	-11	13	19	28	54	26		351
	42	60	16	14	15	75	20	29	39	41		351
	41	59	9	8	83	20	21	30	38	42		351
	40	62	84	-9	33	12	18	31	37	43		351
	45	-98	22	-19	248	23	24	26	36	44		351
	-20	-222	32	343	33	34	36	35	35	45		351
	34	112	19	33	32	31	30	29	40	-9		351
	46	-32	61	47	48	49	50	51	-9	40		351
	351	351	351	351	351	351	351	351	351	351		351

The magic sums of above two examples are as follows:

**First Example**

$$M_{4 \times 4} = 120$$

$$S_{6 \times 6} = 200$$

$$T_{8 \times 8} = 270$$

$$L_{10 \times 10} = 320$$

**Second Example**

$$M_{4 \times 4} = 120$$

$$S_{6 \times 6} = 200$$

$$T_{8 \times 8} = 271$$

$$L_{10 \times 10} = 351$$

Above two examples are considered with magic sums as **even** and **odd** numbers. The magic rectangles sums are:

**First Example**

$$MR_{2 \times 8} = 50 \times 200$$

**Second Example**

$$MR_{2 \times 8} = 80 \times 320$$

**Result 5.7.** Let's consider following magic square of order 10 with *reduced entries*:

$A_{19}+A_{20}+A_{21}+A_{22}+A_{23}-A_4-A_7-A_{11}-A_{15}$	A4	A7	A11	A15	$S-A_{19}-A_{20}-A_{21}-A_{22}-A_{23}$	T-S-A29	A29	L-T-A41	A41
$S-A_{12}-A_5-A_8-A_{16}-A_{19}$	A5	A8	A12	A16	A19	T-S-A30	A30	L-T-A42	A42
$A_4+A_7+A_{11}+A_{15}-A_{20}-A_{21}-A_{22}-A_{23}-A_1-A_2-A_3+A_5+A_8+A_{12}+A_{16}$	$S-A_4-A_5-A_{12}+A_{21}+A_{22}+A_{23}-A_{17}-A_7-A_8-A_{15}-A_{16}-A_{13}-A_9-A_{11}+A_1+A_2+A_3$	A9	A13	A17	A20	T-S-A31	A31	L-T-A43	A43
A1	$S+A_{16}+A_{13}+A_6-A_{19}+A_3-A_{18}-A_{14}-2*A_{21}-A_{22}-A_{23}-A_{20}$	$A_{21}+A_{22}+A_{23}+A_{20}-A_{16}-A_{13}-A_6+A_{19}-A_3$	A14	A18	A21	T-S-A32	A32	L-T-A44	A44
A2	A6	$S+A_{16}+A_{13}-A_9+A_6-A_{19}+A_3-A_{10}-A_{21}-A_{22}-A_{23}-A_{20}-A_7-A_8$	$A_5+A_{14}+A_{10}+2*A_{21}+A_{22}+3*A_{23}+2*A_{20}-A_4+A_8-A_{15}-A_{16}-A_{13}+2*A_9-2*A_6+2*A_{19}-A_{11}-A_2-A_3-S$	$S-A_{19}-A_{20}-A_{21}-A_{22}-2*A_{23}+A_4+A_7+A_{11}+A_{15}-A_5-A_9-A_{14}$	A22	T-S-A33	A33	L-T-A45	A45
A3	$A_{12}+A_{18}+A_{14}+A_{21}+A_{20}+A_{17}+A_7+A_8+A_{15}+A_9-2*A_6+A_{19}+A_{11}-A_2-2*A_3-S$	A10	$2*S+A_4-A_8+A_{15}+A_{16}-2*A_9+2*A_6-2*A_{19}+A_2+A_3-A_{12}-A_5-2*A_{14}-A_{10}-2*A_{21}-A_{22}-3*A_{23}-2*A_{20}$	$A_{19}+A_{20}+A_{21}+A_{22}+2*A_{23}-A_4-A_7-A_{11}+A_5+A_9+A_{14}-2*A_{15}-A_{16}-A_{17}-A_{18}$	A23	$2*S-2*T+A_{29}+A_{30}+A_{31}+A_{32}+A_{33}$	$3*T-3*S-A_{29}-A_{30}-A_{31}-A_{32}-A_{33}$	L-T-A46	A46
T-S-A25	$A_{30}-A_{29}-2*A_9+A_{13}-A_{25}+2*S-2*A_{21}-A_{22}-3*A_{23}-A_{18}-2*A_{19}-3*A_{20}-A_{14}+A_{15}+A_{16}-2*A_{10}+A_2+A_4+A_{11}-A_5+2*A_6-A_8+A_3$	T-S-A26	T-S-A27	T-S-A28	$2*A_{25}+A_{28}+A_{27}+A_{26}-A_{30}+A_{29}+2*A_9-A_{13}-T-S+2*A_{21}+A_{22}+3*A_{23}+A_{18}+2*A_{19}+3*A_{20}+A_{14}-A_{15}-A_{16}+2*A_{10}-A_2-A_4-A_{11}+A_5-2*A_6+A_8-A_3$	(T-S)/2	$(7*S-5*T)/2$	L-T-A47	A47
A25	$A_{29}-A_{30}+2*A_9-A_{13}+T+A_{25}-A_{11}-3*S+2*A_{21}+A_{22}+3*A_{23}+A_{18}+2*A_{19}+3*A_{20}+A_{14}-A_{15}-A_{16}+2*A_{10}-A_2-A_4+A_5-2*A_6+A_8-A_3$	A26	A27	A28	$A_{30}-A_{29}-A_{28}-A_{27}-A_{26}-2*A_9+A_{13}+2*T-2*A_{25}-2*A_{21}-A_{22}-3*A_{23}-A_{18}-2*A_{19}-3*A_{20}-A_{14}+A_{15}+A_{16}-2*A_{10}+A_2+A_4+A_{11}-A_5+2*A_6-A_8+A_3$	$(7*S-5*T)/2$	(T-S)/2	$3*T-3*L+A_{41}+A_{42}+A_{43}+A_{44}+A_{45}+A_{46}+A_{47}$	$4*L-4*T-A_{41}-A_{42}-A_{43}-A_{44}-A_{45}-A_{46}-A_{47}$
L-T-A35	$A_{32}+T-A_{31}+2*S-A_{21}-2*A_{22}-2*A_{23}-A_{26}+A_{18}-A_{19}-A_{20}-A_{14}+2*A_{15}+A_{16}+A_{17}-A_{41}+A_4+A_{42}-A_{35}+A_{27}+A_{11}+A_7-A_5-A_9-2*T$	L-T-A36	L-T-A37	L-T-A38	L-T-A39	L-T-A40	$A_9+2*T-A_{32}+T+A_{36}+A_{31}-2*S+A_{21}+2*A_{22}+2*A_{23}+A_2-6-A_{18}+A_{19}+A_{20}+A_{14}-2*A_{15}-A_{16}-A_{17}+A_{38}+A_{39}+A_{40}+A_{37}+A_{41}-A_4-A_{42}+2*A_{35}-2*L-A_{27}-A_{11}-A_7+A_5$	(L-T)/2	$(9*T-7*L)/2$
A35	$A_9+2*T-A_{32}-2*T+A_{31}-2*S+A_{21}+2*A_{22}+2*A_{23}+A_{26}-A_{18}+A_{19}+A_{20}+A_{14}-2*A_{15}-A_{16}-A_{17}+A_{41}-A_4-A_{42}+A_{35}+L-A_{27}-A_{11}-A_7+A_5$	A36	A37	A38	A39	A40	$A_{32}-2*T-A_{36}-A_{31}+2*S-A_{21}-2*A_{22}-2*A_{23}-A_{26}+A_{18}-A_{19}-A_{20}-A_{14}+2*A_{15}+A_{16}+A_{17}-A_{38}-A_{39}-A_{40}-A_{37}-A_{41}+A_4+A_{42}-2*A_{35}+3*L+A_{27}+A_{11}+A_7-A_5-A_9-2*T$	$(9*T-7*L)/2$	(L-T)/2

### Details:

It is also a **cornered** magic square of order 10 with magic square of order 6 at the upper left corner It includes magic square order 8 also as a **cornered** magic square . The letters S, T and L represents the magic squares of orders 6, 8 and 10 respectively. In order to avoid decimal entries, the magic sums pairs (T, L) and (S, T) should be of same type, i.e., either **even** or **odd** numbers. See the examples below.

**Example 5.10.** Let's consider following two examples:

7a	mgc	10x10 <a href="https://numbers-magic.com/">https://numbers-magic.com/</a> ©IJT										280
	145	17	23	31	39	-105	-17	67	-11	91		280
	-15	19	25	33	41	47	-19	69	-13	93		280
	-19	15	27	35	43	49	-21	71	-15	95		280
	11	-137	143	37	45	51	-23	73	-17	97		280
	13	21	-97	293	-133	53	-25	75	-19	99		280
	15	215	29	-279	115	55	255	-205	-21	101		280
	-9	-323	-11	-13	-15	521	25	25	-23	103		280
	59	373	61	63	65	-471	25	25	439	-359		280
	1	-141	-1	-3	-5	-7	-9	485	40	-80		280
	79	221	81	83	85	87	89	-405	-80	40		280
	280	280	280	280	280	280	280	280	280	280		280

7b	mgc	10x10 <a href="https://inderjtaneja.wordpress.com/">https://inderjtaneja.wordpress.com/</a> ©IJT										281
	145	17	23	31	39	-94	-7	67	-31	91		281
	-4	19	25	33	41	47	-9	69	-33	93		281
	-19	26	27	35	43	49	-11	71	-35	95		281
	11	-126	143	37	45	51	-13	73	-37	97		281
	13	21	-86	282	-122	53	-15	75	-39	99		281
	15	204	29	-257	115	55	235	-175	-41	101		281
	1	-301	-1	-3	-5	489	30	11	-43	103		281
	59	361	61	63	65	-429	11	30	499	-439		281
	-19	-140	-21	-23	-25	-27	-29	524	30	11		281
	79	200	81	83	85	87	89	-464	11	30		281
	281	281	281	281	281	281	281	281	281	281		281

The magic sums of above two examples are as follows:

**First Example**

$$S_{6 \times 6} = 150$$

$$T_{8 \times 8} = 200$$

$$L_{10 \times 10} = 280$$

**Second Example**

$$S_{6 \times 6} = 161$$

$$T_{8 \times 8} = 221$$

$$L_{10 \times 10} = 281$$

Above two examples are considered with magic sums as **even** and **odd** numbers. The magic rectangles sums are:

**First Example**

$$MR_{2 \times 6} = 50 \times 150$$

$$MR_{2 \times 8} = 80 \times 320$$

**Second Example**

$$MR_{2 \times 6} = 60 \times 180$$

$$MR_{2 \times 8} = 60 \times 240$$

**Result 5.8.** Let's consider following magic square of order 10 with *reduced entries*:

$A1-A3-A5-A6-A8+A7$	$A6+A8-2*m+A11+A1-A3-A5$	$2*m$	$2*m+2*A3+2*A5-A7-A11-2*A1$	$A5+2*A3-A7-A11+A10-A9$	$2*m-A9-A5-A10$	$A9-2*A3+A7+A11$	$A9$	$L-4*m-A19$	$A19$
$m+A3+A5+A6+A8-A7-A1$	$3*m+A3+A5-A6-A8-A11-A1$	$(-m)$	$A7-m+A11+2*A1-2*A3-2*A5$	$m+A9-A10+A11-A5-2*A3+A7$	$A9+A5+A10-m$	$m-A9+2*A3-A7-A11$	$m-A9$	$L-4*m-A20$	$A20$
$2*A3+2*A5-A7-A11-A1$	$A1$	$A6+A8+A11-A3-A5$	$2*m+A7-A3-A5-A6-A8$	$A5+2*A3-A7-A11$	$A5$	$2*m+A7+A11-A10-2*A3-2*A5$	$A10$	$3*(4*m-L)+A19+A20+A21+A22+A23+A24+A25$	$4*(L-4*m)-A19-A20-A21-A22-A23-A24-A25$
$A1+m-2*A3-2*A5+A7+A11$	$m-A1$	$m+A3+A5-A6-A8-A11$	$A3+A5+A6+A8-A7-m$	$m+A7+A11-A5-2*A3$	$m-A5$	$2*A3+2*A5-A7-A11+A10-m$	$m-A10$	$L-4*m-A21$	$A21$
$A3+A2-A7-A11+A4$	$2*m-A3-A2-A4$	$A7+A11-A3$	$A3$	$2*m-A6-A8-A11$	$A6+A8-A7$	$A7$	$A11$	$L-4*m-A22$	$A22$
$m+A7+A11-A4-A3-A2$	$A3+A2+A4-m$	$m+A3-A7-A11$	$m-A3$	$A6+A8+A11-m$	$m+A7-A6-A8$	$m-A7$	$m-A11$	$L-4*m-A23$	$A23$
$A2+2*A3-A7-A11$	$A2$	$2*m+A7+A11-2*A2-A4-2*A3$	$A4$	$A6+A11-A7$	$A6$	$A8$	$2*m+A7-2*A6-A8-A11$	$L-4*m-A24$	$A24$
$m+A7+A11-A2-2*A3$	$m-A2$	$2*A3-A7-A11+2*A2+A4-m$	$m-A4$	$m+A7-A11-A6$	$m-A6$	$m-A8$	$2*A6+A8+A11-A7-m$	$L-4*m-A25$	$A25$
$L-4*m-A13$	$7*m-2*A10-2*A2-2*A4-A19+A20-A13+A11+3*A7-2*A5-2*A6-2*A8-4*A3$	$m-2*L+2*A13-A20+A19+2*A4+2*A6+2*A8-3*A7+4*A3+2*A5+2*A10+2*A2-A11+A14+A15+A16+A17+A18$	$L-4*m-A14$	$L-4*m-A15$	$L-4*m-A16$	$L-4*m-A17$	$L-4*m-A18$	$(L-4*m)/2$	$(9*4*m-7*L)/2$
$A13$	$L-11*m+2*A10+2*A2+2*A4+A19-A20+A13-A11-3*A7+2*A5+2*A6+2*A8+4*A3$	$3*L-5*m-2*A13+A20-A19-2*A4-2*A6-2*A8+3*A7-4*A3-2*A5-2*A10-2*A2+A11-A14-A15-A16-A17-A18$	$A14$	$A15$	$A16$	$A17$	$A18$	$(9*4*m-7*L)/2$	$(L-4*m)/2$

### Details:

It is also a **cornered** magic square of order 10 with striped magic square of order 8 at the upper left corner. The letters T and L represents the magic squares of orders 8 and 10 respectively. Here the magic sum of order 8 is always an even numbers. To get the integer entries, the magic sum of order 10 should also be an even number, otherwise some of the entries may be decimal numbers. See the examples below.

**Example 5.11.** Let's consider following two examples:



8a	mgc	10x10 <a href="https://numbers-magic.com/">https://numbers-magic.com/</a> ©IJT										200
	-34	-22	60	56	4	6	31	19	51	29		200
	64	52	-30	-26	26	24	-1	11	50	30		200
	7	11	27	15	3	15	22	20	-16	96		200
	23	19	3	15	27	15	8	10	49	31		200
	1	21	25	13	5	17	17	21	48	32		200
	29	9	5	17	25	13	13	9	47	33		200
	0	12	34	14	20	16	18	6	46	34		200
	30	18	-4	16	10	14	12	24	45	35		200
	57	18	-25	56	55	54	53	52	40	-160		200
	23	62	105	24	25	26	27	28	-160	40		200
	200	200	200	200	200	200	200	200	200	200		200

8b	mgc	10x10 <a href="https://inderjtaneja.wordpress.com/">https://inderjtaneja.wordpress.com/</a> ©IJT										221
	-34	-22	60	56	4	6	31	19	72	29		221
	64	52	-30	-26	26	24	-1	11	71	30		221
	7	11	27	15	3	15	22	20	-79	180		221
	23	19	3	15	27	15	8	10	70	31		221
	1	21	25	13	5	17	17	21	69	32		221
	29	9	5	17	25	13	13	9	68	33		221
	0	12	34	14	20	16	18	6	67	34		221
	30	18	-4	16	10	14	12	24	66	35		221
	78	18	-67	77	76	75	74	73	50.5	-233.5		221
	23	83	168	24	25	26	27	28	-233.5	50.5		221
	221	221	221	221	221	221	221	221	221	221		221

The magic sums of above two examples are as follows:

**First Example**

$$T_{8 \times 8} = 120$$

$$L_{10 \times 10} = 200$$

**Second Example**

$$T_{8 \times 8} = 120$$

$$L_{10 \times 10} = 221$$

Above two examples are considered with magic sums as **even** and **odd** numbers. The magic rectangles sums are:

**First Example**

$$MR_{2 \times 4} = 30 \times 60$$

$$MR_{2 \times 8} = 80 \times 320$$

**Second Example**

$$MR_{2 \times 4} = 30 \times 60$$

$$MR_{2 \times 8} = 101 \times 404$$

Here in the second example we have some entries as **decimal** values. It is due to the fact that magic square of order 8 is always an **even** number and the total magic sum of order 10 is an **odd** number.

**Result 5.9.** Let's consider following magic square of order 10 with **reduced entries**:



$2*M-2*A4-2*A7-A12+A3+A6$	$3*A4+3*A7-A15-A2+A12-A3-A6-2*M$	$A2+A14-A16-4*A4-4*A7+A3+A6-A1+4*M+A15$	$A16+3*A4-3*M+3*A7-A3-A6+A1-A14$	A3	$2*M-A9-A4-A7-A5-A3$	$A5+A8-A16+A3+A6-A1+A9$	$A16+A4+A7-A3-A6+A1-A8-M$	L-2*M-A25	A25
A2	$A4+A7-A15-M$	A1	$2*M-A4-A7+A15-A1-A2$	A5	$M-A9-A4-A7$	$A16+2*A4-2*M+2*A7+A1-A10$	$A9-A4-A7+2*M-A16-A1+A10-A5$	L-2*M-A26	A26
$2*A4+2*A7-A13-M-A14+A16-A3-A6+A1-A2$	$4*M-3*A4-3*A7-A12+A3+A6-A13-A1$	$2*A4+2*A7+A12-A3-A6+A13+A15-2*M$	$A2+A13+A14-A16-A4-A7+A3+A6-A15$	$2*A4-M-A8+A16+A7-A3-A6+A1-A5$	$A3-A4+2*M-A16-2*A7-A1+A10$	$A9+A7-A3$	$A8+A3+A6-A10+A5-A9-A4$	L-2*M-A27	A27
$A12+A13+A14-A16-A1$	$A1-A4-A7+A13+2*A15+A2$	$2*A4+2*A7-A12-A13-A14-M+A16-2*A15-A2$	$2*M-A4-A7-A13$	$2*M-2*A4+A8-A16-A7+A6-A1$	$A16+3*A4-4*M+4*A7+A1+2*A9+A5-A10$	$3*M-A8-3*A7-A6-2*A9+A10-A5-2*A4$	A4	L-2*M-A28	A28
A6	$A9+M-A11-A6$	$A11-A8-A9$	A8	A12	$A15+M-A17-A12$	$A17-A14-A15$	A14	L-2*M-A29	A29
A11	A9	A10	$M-A9-A10-A11$	A17	A15	A16	$M-A15-A16-A17$	L-2*M-A30	A30
$A7+A8-A11$	$A6+A7-A10$	$M-A6-A7-A9$	$A9+A10+A11-A7-A8$	$A13+A14-A17$	$A12+A13-A16$	$M-A12-A13-A15$	$A15+A16+A17-A13-A14$	L-2*M-A31	A31
$M-A6-A7-A8$	$A10-A7-2*A9+A11$	$A6+A7+A8+2*A9-A10-A11$	A7	$M-A12-A13-A14$	$A16-A13-2*A15+A17$	$A12+A13+A14+2*A15-A16-A17$	A13	$6*M-3*L+A25+A26+A27+A28+A29+A30+A31$	$4*L-8*M-A25-A26-A27-A28-A29-A30-A31$
L-2*M-A19	$3*A4+A26-A19+A12+3*A7-A6-A3-2*M-A15-A25$	L-2*M-A20	L-2*M-A21	L-2*M-A22	L-2*M-A23	L-2*M-A24	$6*M-2*L+2*A19+A15+A25-3*A4-A26-A12-3*A7+A6+A3+A20+A21+A22+A23+A24$	$(L-2*M)/2$	$(9*2*M-7*L)/2$
A19	$A15+A25-3*A4-A26+A19+L-A12-3*A7+A6+A3$	A20	A21	A22	A23	A24	$3*L-8*M-2*A19-A15-A25+3*A4+A26+A12+3*A7-A6-A3-A20-A21-A22-A23-A24$	$(9*2*M-7*L)/2$	$(L-2*M)/2$

**Details:**

It is also a **cornered** magic square of order 10 with **pandiagonal** magic square of order 8 at the left-upper corner. The letters T and L represents the magic squares of orders 8 and 10 respectively. The magic sum of order 8 is always an even numbers as it is formed by four equal sums magic squares of order 4. To get the integer entries, the magic sum of order 10 should also be an **even** number, otherwise some of the entries may be decimal numbers. See the examples below.

**Example 5.12.** Let's consider following two examples:

9a	mgc	10x10 <a href="https://numbers-magic.com/">https://numbers-magic.com/</a> ©IJT										220
	59	-83	241	-127	19	33	77	-39	-67	107		220
	15	-99	11	163	27	-11	-29	103	-71	111		220
	-79	111	67	-9	-43	71	59	3	-75	115		220
	95	161	-229	63	87	-3	-17	23	-79	119		220
	31	51	-31	39	55	27	-55	63	-83	123		220
	51	43	47	-51	75	67	71	-123	-87	127		220
	23	19	-19	67	47	43	-91	91	-91	131		220
	-15	-23	93	35	-87	-47	165	59	713	-673		220
	-43	-147	-47	-51	-55	-59	-63	625	20	40		220
	83	187	87	91	95	99	103	-585	40	20		220
	220	220	220	220	220	220	220	220	220	220		220

9b	mgc	10x10 <a href="https://inderjtaneja.wordpress.com/">https://inderjtaneja.wordpress.com/</a> ©IJT										261
	59	-83	241	-127	19	33	77	-39	-26	107		261
	15	-99	11	163	27	-11	-29	103	-30	111		261
	-79	111	67	-9	-43	71	59	3	-34	115		261
	95	161	-229	63	87	-3	-17	23	-38	119		261
	31	51	-31	39	55	27	-55	63	-42	123		261
	51	43	47	-51	75	67	71	-123	-46	127		261
	23	19	-19	67	47	43	-91	91	-50	131		261
	-15	-23	93	35	-87	-47	165	59	590	-509		261
	-2	-147	-6	-10	-14	-18	-22	543	40.5	-103.5		261
	83	228	87	91	95	99	103	-462	-103.5	40.5		261
	261	261	261	261	261	261	261	261	261	261		261

The magic sums of above two examples are as follows:

**First Example**

$$M_{4 \times 4} = 90$$

$$T_{8 \times 8} = 180$$

$$L_{10 \times 10} = 220$$

**Second Example**

$$M_{4 \times 4} = 90$$

$$T_{8 \times 8} = 180$$

$$L_{10 \times 10} = 261$$

Above two examples are considered with magic sums as **even** and **odd** numbers. The magic rectangles sums are:

**First Example**

$$MR_{2 \times 8} = 40 \times 160$$

**Second Example**

$$MR_{2 \times 8} = 81 \times 324$$

Here in the second example we have some entries as **decimal** values. It is due to the fact that magic square of order 8 is always an **even** number and the magic sum of order 10 is an **odd** number.

## 5.2 Reduced Entries Pandiagonal Magic Squares of Order 10

Below is a result for **reduced entries algebraic pandiagonal** magic square of order 10.

**Result 5.10.** *Let's consider following magic square of order 10 with **reduced entries**:*

$A7-A23+A15$	$A1$	$S-A7+A23-A15-A1-A2-A3$	$A2$	$A3$	$A7$	$A8$	$S-A7-A8-A9-A10$	$A9$	$A10$
$A21-A28+A13-A1+A6$	$A4$	$A7-A23+A15+A1+A2+A3-A4-A5-A21+A28-A13$	$A5$	$S-A7+A23-A15-A2-A3-A6$	$A14-A8+A13$	$A11$	$A7+A8+A9+A10-A11-A12-A13$	$A12$	$S-A7-A9-A10-A14$
$A1+A2+A3-A4-A21+A28-A13$	$S-A7+A23-A15-A1-A2-A3+A5-A6$	$A21-A28+A13$	$A21-A28+A13-A1-A2+A4+A6$	$A7-A23+A15+A1+A2-A5-A21+A28-A13$	$A8+A9+A10-A11-A13$	$S-A7-A8-A9-A10+A12-A14$	$A13$	$A13-A8-A9+A11+A14$	$A7+A8+A9-A12-A13$
$A21-A28+A13-A2+A4$	$A7-A23+A15-A5+A6$	$A1+A2-A21+A28-A13$	$S-A7+A23-A15-A4-A6-A21+A28-A13$	$A21-A28+A13-A1+A5$	$A13-A9+A11$	$A7-A12+A14$	$A8+A9-A13$	$S-A7-A11-A13-A14$	$A13-A8+A12$
$S-A7+A23-A15-A3-A6-A21+A28-A13$	$A2+A3-A4$	$A21-A28+A13-A1-A2+A4+A5$	$A7-A23+A15+A1-A5$	$A6$	$S-A7-A10-A13-A14$	$A9+A10-A11$	$A13-A8-A9+A11+A12$	$A7+A8-A12$	$A14$
$A15$	$A16$	$S-A15-A16-A17-A18$	$A17$	$A18$	$A23$	$A24$	$S-A23-A24-A25+A3-A18-A10$	$A25$	$A18+A10-A3$
$A22-A16+A21$	$A19$	$A15+A16+A17+A18-A19-A20-A21$	$A20$	$S-A15-A17-A18-A22$	$A22+A14-A6-A24+A28$	$A26$	$A23+A24+A25-A3+A18+A10-A26-A27-A28$	$A27$	$S-A23-A25+A3-A18-A10-A22-A14+A6$
$A16+A17+A18-A19-A21$	$S-A15-A16-A17-A18+A20-A22$	$A21$	$A21-A16-A17+A19+A22$	$A15+A16+A17-A20-A21$	$A24+A25-A3+A18+A10-A26-A28$	$S-A23-A24-A25+A3-A18-A10+A27-A22-A14+A6$	$A28$	$A28-A24-A25+A26+A22+A14-A6$	$A23+A24+A25-A27-A28$
$A21-A17+A19$	$A15-A20+A22$	$A16+A17-A21$	$S-A15-A19-A21-A22$	$A21-A16+A20$	$A28-A25+A26$	$A23-A27+A22+A14-A6$	$A24+A25-A28$	$S-A23-A26-A28-A22-A14+A6$	$A28-A24+A27$
$S-A15-A18-A21-A22$	$A17+A18-A19$	$A21-A16-A17+A19+A20$	$A15+A16-A20$	$A22$	$S-A23+A3-A18-A10-A28-A22-A14+A6$	$A25-A3+A18+A10-A26$	$A28-A24-A25+A26+A27$	$A23+A24-A27$	$A22+A14-A6$

### Details:

It a **pandiagonal** magic square of order 10 divided in four equal sums blocks of order 5. These blocks are **pandiagonal** magic squares of order 5. The magic sum of order 10 is represented by letter  $T = 2S$ , where  $S$  is the sum of magic square of order 5. In this case we always have even number magic sums of order 10. See below an example.

**Example 5.13.** Let's consider the following example:

	pan	180	180	180	180	180	180	180	180	180	180
180	9	11	45	12	13	17	18	16	19	20	180
180	21	14	0	15	40	29	21	8	22	10	180
180	6	44	16	23	1	13	14	23	31	9	180
180	18	10	7	35	20	25	19	14	5	27	180
180	36	11	22	5	16	6	18	29	13	24	180
180	25	26	-16	27	28	33	34	-47	35	35	180
180	37	29	16	30	-22	44	36	26	37	-53	180
180	21	-18	31	39	17	30	-50	38	45	27	180
180	33	27	22	-27	35	39	36	31	-57	41	180
	-26	26	37	21	32	-56	34	42	30	40	180
	180	180	180	180	180	180	180	180	180	180	180

It is **pandiagonal** magic square of order 10 based on the Result 5.10 with magic sums  $T_{10 \times 10} = 180$  and  $S_{5 \times 5} = 90$ . The four equal sums **pandiagonal** magic square of order 5 are given by

	pan	90	90	90	90	90			pan	90	90	90	90	90	
90	9	11	45	12	13	90		90	17	18	16	19	20	90	
90	21	14	0	15	40	90		90	29	21	8	22	10	90	
90	6	44	16	23	1	90		90	13	14	23	31	9	90	
90	18	10	7	35	20	90		90	25	19	14	5	27	90	
	36	11	22	5	16	90			6	18	29	13	24	90	
	90	90	90	90	90	90			90	90	90	90	90	90	
	pan	90	90	90	90	90			pan	90	90	90	90	90	
90	25	26	-16	27	28	90		90	33	34	-47	35	35	90	
90	37	29	16	30	-22	90		90	44	36	26	37	-53	90	
90	21	-18	31	39	17	90		90	30	-50	38	45	27	90	
90	33	27	22	-27	35	90		90	39	36	31	-57	41	90	
	-26	26	37	21	32	90			-56	34	42	30	40	90	
	90	90	90	90	90	90			90	90	90	90	90	90	

**Remark 3.** Above we have given a *pandiagonal* magic square of order 10 for *reduced entries* with four equal sums *pandiagonal* magic square of order 5. The general procedure without use of *pandiagonal* magic squares of order 5 shall be given later on in another work.

### 5.3 Reduced Entries Semi-Magic Squares of Order 10

Below are 6 results for **reduced entries semi-magic** squares based on Examples 5.1, 5.2 and 5.3.

**Result 5.11.** Let's consider following *semi-magic* square of order 10 with *reduced entries*:



$2*(L-S)/2-A42-A28-A35$	$2*A35+A42+A41+A28-A29-2*A22+(5*S-3*L)/2-A21$	A23	A24	A25	A26	$3*(L-S)/2-A23-A24-A25-A26-A27$	A27	$A29+2*A22-A28+A21-A35-A41$	A28
A21	A22	$(L-S)/2-A23$	$(L-S)/2-A24$	$(L-S)/2-A25$	$(L-S)/2-A26$	$A23+A24+A25+A26+A27-2*(L-S)/2$	$(L-S)/2-A27$	$(3*S-L)/2-A21-A22-A29$	A29
$3*(L-S)/2-A16-A17-A18-A19-A20$	$A16+A17+A18+A19+A20-2*(L-S)/2$	$4*S-6*M+A12$	$S-M+2*A14+2*A13-4*A12$	$2*A12+9*M-6*S-A14-A13$	S-M-A14	S-M-A13	A12	$(L-S)/2-A36$	A36
A16	$(L-S)/2-A16$	$4*M-3*S+A9+A10+A11$	A1	$A4+M-A6-A1$	$A6-A3-A4$	A3	$4*S-5*M-A9-A10-A11$	$(L-S)/2-A37$	A37
A17	$(L-S)/2-A17$	S-M-A9	A6	A4	A5	$M-A4-A5-A6$	A9	$S-L+A36+A37+A38+A39+A40$	$3*(L-S)/2-A36-A37-A38-A39-A40$
A18	$(L-S)/2-A18$	S-M-A10	$A2+A3-A6$	$A1+A2-A5$	$M-A1-A2-A4$	$A4+A5+A6-A2-A3$	A10	$(L-S)/2-A38$	A38
A19	$(L-S)/2-A19$	S--M-A11	$M-A1-A2-A3$	$A5-A2-2*A4+A6$	$A1+A2+A3+2*A4-A5-A6$	A2	A11	$(L-S)/2-A39$	A39
A20	$(L-S)/2-A20$	$5*M-3*S-A12$	$4*A12-2*A14-2*A13$	$7*(L-2*(L-S)/2)+A14+A13-2*A12-10*M$	A14	A13	$5*M-A12-3*S$	$(L-S)/2-A40$	A40
$(5*S-3*L)/2+A42+A28-A21$	$(3*L-5*S)/2-A28+A21+A22-A35+A29$	$(L-S)/2-A30$	$A30+A31+A32+A33+A34-2*(L-S)/2$	$(L-S)/2-A31$	$(L-S)/2-A32$	$(L-S)/2-A33$	$(L-S)/2-A34$	$A35+A28-A22$	$(3*S-L)/2-A28-A29-A42$
A35	$(3*S-L)/2-A35-A42-A41$	A30	$3*(L-S)/2-A30-A31-A32-A33-A34$	A31	A32	A33	A34	A41	A42

### Details:

It is a **double-digit bordered semi-magic** square of order 10 embedded with **single-digit bordered semi-magic** squares of orders 6. These are **semi-magic** only at one diagonal. In order to bring it as a magic square we need some condition. It is as given below

$$M = \frac{2}{3} \times S, \quad (4)$$

where M and S are magic sums of orders 4 and 6 respectively.



Below are three examples based on the Result 5.11. First example is of **semi-magic** squares and the second example as a magic square obtained by applying the conditions given in (4)

**Example 5.14.** Let's consider a following **semi-magic** square based on the Result 5.11:

1a	semi	M=(2/3)*S									180
	-75	147	33	34	35	36	-85	37	0	38	200
	31	32	-3	-4	-5	-6	115	-7	8	39	200
	-50	80	42	56	-33	26	27	22	-16	46	200
	26	4	0	11	77	-11	13	50	-17	47	200
	27	3	31	16	14	15	45	19	180	-150	200
	28	2	30	9	8	53	20	20	-18	48	200
	29	1	29	54	-9	33	12	21	-19	49	200
	30	0	8	-6	83	24	23	8	-20	50	200
	109	-31	-10	150	-11	-12	-13	-14	51	-19	200
	45	-38	40	-120	41	42	43	44	51	52	200
	200	200	200	200	200	200	200	200	200	200	200

The **magic** and **semi-magic** sums of above magic square are  $M_{4 \times 4} = 90$ ,  $S_{Sm_{6 \times 6}} = 140$  and  $L_{Sm_{10 \times 10}} = 200$ . There are four magic rectangles with equal magic sums  $MR_{2 \times 6} = 30 \times 90$ .

**Example 5.15.** Let's consider a following magic square with **even** number magic sum based on the Result 5.11:

1b	mgc	M=(2/3)*S									260
	-55	157	33	34	35	36	-55	37	0	38	260
	31	32	7	6	5	4	95	3	38	39	260
	-20	60	22	66	-3	36	37	22	-6	46	260
	26	14	0	11	107	-11	13	60	-7	47	260
	27	13	41	16	14	15	75	19	160	-120	260
	28	12	40	9	8	83	20	20	-8	48	260
	29	11	39	84	-9	33	12	21	-9	49	260
	30	10	38	-6	63	24	23	38	-10	50	260
	119	-41	0	130	-1	-2	-3	-4	51	11	260
	45	-8	40	-90	41	42	43	44	51	52	260
	260	260	260	260	260	260	260	260	260	260	260

The **magic** and **semi-magic sums** of above magic square are  $M_{4 \times 4} = 120$ ,  $S_{6 \times 6} = 180$  and  $L_{10 \times 10} = 260$ . It is a magic square as it satisfies the conditions given in (4), i.e.,  $M = \frac{2}{3} \times S$ . Moreover, the magic rectangles sums is  $MR_{2 \times 6} = 40 \times 120$ .

**Example 5.16.** Let's consider a following magic square with **odd** number magic sum based on the Result 5.11:

1c	mgc	M=(2/3)*S									203
	-55	100	33	34	35	36	-55	37	0	38	203
	31	32	7	6	5	4	95	3	-19	39	203
	-20	60	22	47	-3	17	18	22	-6	46	203
	26	14	19	11	69	-11	13	22	-7	47	203
	27	13	22	16	14	15	37	19	160	-120	203
	28	12	21	9	8	45	20	20	-8	48	203
	29	11	20	46	-9	33	12	21	-9	49	203
	30	10	19	-6	44	24	23	19	-10	50	203
	62	16	0	130	-1	-2	-3	-4	51	-46	203
	45	-65	40	-90	41	42	43	44	51	52	203
	203	203	203	203	203	203	203	203	203	203	203

The **magic** and **semi-magic** sums of above magic square are  $M_{4 \times 4} = 82$ ,  $S_{6 \times 6} = 123$  and  $L_{10 \times 10} = 203$ . It is a magic square as it satisfies the conditions given in (4), i.e.,  $M = \frac{2}{3} \times S$ . Moreover, the magic rectangles sums is  $MR_{2 \times 6} = 40 \times 120$

**Result 5.12.** Let's consider following magic square of order 10 with *reduced entries*:

$10^*T+A43-9^*L$	$27^*L-2^*A43-28^*T-A42-A41-A40-A38-A37-A36-A34-A35-A32-A33-A29-A30-A31-A28-A39$	A36	A37	A38	A39	A40	A41	A42	$A34+A35+A43+A32+A33+A29+A30+A31+A28+18^*T-17^*L$
A28-T	A18	A19	$2^*(T-S)/2-A15-A16-A17$	A15	A16	A17	$2^*S-T+A18-A19+A23$	$T-S-A23-2^*A18$	L-A28
A29	$2^*A13+A24+A14+2^*A18-2^*(T-S)/2-A19$	$T-S-2^*A18-A13$	$A15+A16+A17-(T-S)/2$	$(T-S)/2-A15$	$(T-S)/2-A16$	$(T-S)/2-A17$	$T-S-A23-A13-A24-A18+A19$	$2^*S-T+A18+A23-A14$	L-T-A29
A30	$2^*(T-S)/2-A7-A8-A9$	$A7+A8+A9-(T-S)/2$	A1	$A4+S-A6-A1$	$A6-A3-A4$	A3	$A10+A11+A12-(T-S)/2$	$T-S-A10-A11-A12$	L-T-A30
A31	A7	$(T-S)/2-A7$	A6	A4	A5	$S-A4-A5-A6$	$(T-S)/2-A10$	A10	L-T-A31
A32	A8	$(T-S)/2-A8$	$A2+A3-A6$	$A1+A2-A5$	$S-A1-A2-A4$	$A4+A5+A6-A2-A3$	$(T-S)/2-A11$	A11	L-T-A32
A33	A9	$(T-S)/2-A9$	$S-A1-A2-A3$	$A5-A2-2^*A4+A6$	$A1+A2+A3+2^*A4-A5-A6$	A2	$(T-S)/2-A12$	A12	L-T-A33
A34	$T-2^*A13-A14+A19-3^*A18-A23-A24$	$A23+3^*A18+A24-A19+A13-2^*(T-S)/2$	$A20+A21+A22-(T-S)/2$	$(T-S)/2-A20$	$(T-S)/2-A21$	$(T-S)/2-A22$	A13	A14	L-T-A34
A35	A23	$S-A23-A18-A24$	$T-S-A20-A21-A22$	A20	A21	A22	A24	A18	L-T-A35
$10^*L-9^*T-A28-A29-A30-A31-A32-A33-A34-A35-A43$	$27^*T+2^*A43+A42+A41+A40+A38+A37+A36+A34+A35+A32+A33+A29+A30+A31+A28-26^*L+A39$	L-T-A36	L-T-A37	L-T-A38	L-T-A39	L-T-A40	L-T-A41	L-T-A42	$10^*L-11^*T-A43$

*Details:*

It is a *single-digit bordered semi-magic* square of order 10 embedded with *double-digit bordered magic* square of orders 8 . These are *semi-magic* only at one diagonal. In order to bring it as a magic square we need some condition. It is given as

$$T = \frac{4}{5} \times L \quad (5)$$

where  $T$  and  $L$  are magic sums of orders 8 and 10 respectively.

Below are three examples based on the Result 5.12. First example is of **semi-magic** squares and the second example as a magic square obtained by applying the conditions given in (5).

**Example 5.17.** Let's consider a following **semi-magic** square based on the Result 5.12:

2a	semi	T=(4/5)*L									160
	-185	-195	81	83	85	87	89	91	93	-9	220
	-105	45	47	-43	39	41	43	63	-65	155	220
	67	127	-45	83	1	-1	-3	-65	73	-17	220
	69	5	35	11	75	-11	15	53	-13	-19	220
	71	23	17	21	17	19	33	11	29	-21	220
	73	25	15	7	5	49	29	9	31	-23	220
	75	27	13	51	-7	33	13	7	33	-25	220
	77	-137	155	113	-9	-11	-13	35	37	-27	220
	79	55	-67	-73	49	51	53	57	45	-29	220
	-1	245	-31	-33	-35	-37	-39	-41	-43	235	220
	220	220	220	220	220	220	220	220	220	220	220

The **magic** and **semi-magic** sums of above magic square are  $M_{4 \times 4} = 90$ ,  $T_{8 \times 8} = 170$  and  $L_{Sm_{10 \times 10}} = 220$ . There are four magic rectangles with equal magic sums  $MR_{2 \times 4} = 40 \times 80$ .

**Example 5.18.** Let's consider a following magic square with **even** number magic sum based on the Result 5.12:

2b	mgc	T=(4/5)*L									220
	-125	-363	81	83	85	87	89	91	93	99	220
	-111	45	47	-53	39	41	43	89	-75	155	220
	67	137	-55	88	-4	-6	-8	-75	99	-23	220
	69	-5	40	11	91	-11	15	58	-23	-25	220
	71	23	12	21	17	19	49	6	29	-27	220
	73	25	10	7	5	65	29	4	31	-29	220
	75	27	8	67	-7	33	13	2	33	-31	220
	77	-131	165	118	-14	-16	-18	35	37	-33	220
	79	55	-51	-83	49	51	53	57	45	-35	220
	-55	407	-37	-39	-41	-43	-45	-47	-49	169	220
	220	220	220	220	220	220	220	220	220	220	220

The **magic sums** of above magic square are  $M_{4 \times 4} = 106$ ,  $T_{8 \times 8} = 176$  and  $L_{10 \times 10} = 220$ . It is a magic square as it satisfies the conditions given in (5), i.e.,  $T = \frac{4}{5} \times L$ . Moreover, there are four magic rectangles with equal magic sums  $MR_{2 \times 4} = 35 \times 70$ .

**Example 5.19.** Let's consider a following magic square with **odd** number magic sum based on the Result 5.12:

2c	mgc	T=(4/5)*L									315
	-220	74	81	83	85	87	89	91	93	-148	315
	-187	45	47	-23	39	41	43	105	-45	250	315
	67	107	-25	73	11	9	7	-45	115	-4	315
	69	25	25	11	137	-11	15	43	7	-6	315
	71	23	27	21	17	19	95	21	29	-8	315
	73	25	25	7	5	111	29	19	31	-10	315
	75	27	23	113	-7	33	13	17	33	-12	315
	77	-55	135	103	1	-1	-3	35	37	-14	315
	79	55	-5	-53	49	51	53	57	45	-16	315
	211	-11	-18	-20	-22	-24	-26	-28	-30	283	315
	315	315	315	315	315	315	315	315	315	315	315

The **magic sums** of above magic square are  $M_{4 \times 4} = 152$ ,  $T_{8 \times 8} = 252$  and  $L_{10 \times 10} = 315$ . It is a magic square as it satisfies the conditions given in (5), i.e.,  $T = \frac{4}{5} \times L$ . Moreover, there are four magic rectangles with equal magic sums  $MR_{2 \times 4} = 50 \times 100$ .

**Result 5.13.** Let's consider following mgic squares of order 10 with *reduced entries*:

10*T-9*L	27*L-28*T-A36-A35-A34-A32-A31-A30-A28-A29-A26-A27-A23-A24-A25-A22-A33	A30	A31	A32	A33	A34	A35	A36	18*T-17*L+A28+A29+A26+A27+A23+A24+A25+A22
A22-T	A1	A4+M-A6-A1	A6-A3-A4	A3	S-M-A10	A10	T-S-A17	A17	L-A22
A23	A6	A4	A5	M-A4-A5-A6	A10+A11+A12-S+M	2*S-2*M-A10-A11-A12	T-4*S-A17+A1+2*M+2*A9+2*A8+3*A4	A17-A1-2*M+3*S-2*A9-2*A8-3*A4	L-T-A23
A24	A2+A3-A6	A1+A2-A5	M-A1-A2-A4	A4+A5+A6-A2-A3	S-M-A11	A11	2*A17+A19+A20-A1-2*M+5*S-2*T-2*A9-2*A8-3*A4+A18	3*T-6*S-A19-A20-2*A17+A1+2*M+2*A9+2*A8+3*A4-A18	L-T-A24
A25	M-A1-A2-A3	A5-A2-2*A4+A6	A1+A2+A3+2*A4-A5-A6	A2	S-M-A12	A12	T-S-A18	A18	L-T-A25
A26	S-M-A8	2*S-A8-A1-3*A4-2*A10-A11-A12-M	2*A8+A1+A9+3*A4+2*A10+A11+A12+M-2*S	S-M-A9	(S-M)/2	(5*M-3*S)/2	T-S-A19	A19	L-T-A26
A27	A8	A8+A1+3*A4+2*A10+A11+A12-S	3*S-2*A8-A1-A9-3*A4-2*A10-A11-A12-2*M	A9	(5*M-3*S)/2	(S-M)/2	T-S-A20	A20	L-T-A27
A28	T+2*S-3*M-2*A9-2*A8	2*A10+2*A12-M+2*S-2*A9-2*A8	A16-4*S+4*M-T+4*A9+4*A8+A15+A14-2*A12-2*A10	T-S-A14	T-S-A15	T-S-A16	(T-S)/2	(7*S-5*T)/2	L-T-A28
a43	2*A9+2*A8-3*S+3*M	2*A9+2*A8+M+T-3*S-2*A10-2*A12	2*T+3*S-A15-A14-4*M-4*A9-4*A8+2*A12+2*A10-A16	A14	A15	A16	(7*S-5*T)/2	(T-S)/2	L-T-A29
10*L-9*T-A22-A23-A24-A25-A26-A27-A28-A29	27*T+A36+A35+A34+A33+A32+A31+A30+A28+A29+A26+A27+A23+A24+A25+A22-26*L	L-T-A30	L-T-A31	L-T-A32	L-T-A33	L-T-A34	L-T-A35	L-T-A36	10*L-11*T

*Details:*

It is a *single-digit bordered semi-magic square* of order 10 embedded with *cornered magic square* of order 8. The inner block is again



a **cornered** magic square of order 6 with magic square of order 4 at the left upper corner. In order to bring it as a magic square we the same condition as given in (5).

Below are three examples based on the Result 5.13. First example is of **semi-magic** square and the second example as a magic square obtained by applying the condition given in (5)

**Example 5.20.** Let's consider a following **semi-magic** square based on the Result 5.13:

3a	semi	T=(4/5)*L									320
	-160	145	50	51	52	53	54	55	56	-116	240
	-158	21	67	-21	23	0	30	43	37	198	240
	43	26	24	25	15	63	-33	70	10	-3	240
	44	19	18	23	30	-1	31	4	76	-4	240
	45	24	-19	63	22	-2	32	42	38	-5	240
	46	2	-94	151	1	15	45	41	39	-6	240
	47	28	124	-121	29	45	15	40	40	-7	240
	48	56	160	-111	46	45	44	40	-80	-8	240
	49	24	-80	191	34	35	36	-80	40	-9	240
	236	-105	-10	-11	-12	-13	-14	-15	-16	200	240
	240	240	240	240	240	240	240	240	240	240	240

The **magic** and **semi-magic** sums of above **semi-magic** square are  $M_{4 \times 4} = 90$ ,  $S_{6 \times 6} = 120$ ,  $T_{8 \times 8} = 200$  and  $L_{Sm_{12 \times 12}} = 240$ . The magic rectangle sums are given as  $MR_{2 \times 4} = 30 \times 60$  and  $MR_{2 \times 6} = 80 \times 240$ .

**Example 5.21.** Let's consider a following magic square with **even** number magic sum based on the Result 5.13:

3b	mgc	T=(4/5)*L									280
	-280	553	50	51	52	53	54	55	56	-364	280
	-182	21	87	-21	23	10	30	37	37	238	280
	43	26	24	25	35	53	-13	14	60	13	280
	44	19	18	43	30	9	31	66	8	12	280
	45	44	-19	63	22	8	32	36	38	11	280
	46	12	-54	111	11	20	50	35	39	10	280
	47	28	94	-71	29	50	20	34	40	9	280
	48	80	200	-175	40	39	38	37	-35	8	280
	49	-6	-126	249	34	35	36	-35	37	7	280
	420	-497	6	5	4	3	2	1	0	336	280
	280	280	280	280	280	280	280	280	280	280	280

The **magic sums** of above magic square are  $M_{4 \times 4} = 110$ ,  $S_{6 \times 6} = 150$ ,  $T_{8 \times 8} = 224$  and  $L_{10 \times 10} = 280$ . It is a magic square as it satisfies the condition given in (5), i.e., i.e.,  $T = \frac{4}{5} \times L$ . Moreover, the magic sums of magic rectangles are  $MR_{2 \times 4} = 30 \times 60$  and  $MR_{2 \times 4} = 74 \times 222$ .

**Example 5.22.** Let's consider a following magic square with **odd** number magic sum based on the Result 5.13:

3c	mgc	T=(4/5)*L									305
	-305	668	50	51	52	53	54	55	56	-429	305
	-202	21	87	-21	23	10	30	57	37	263	305
	43	26	24	25	35	53	-13	34	60	18	305
	44	19	18	43	30	9	31	26	68	17	305
	45	44	-19	63	22	8	32	56	38	16	305
	46	12	-54	111	11	20	50	55	39	15	305
	47	28	94	-71	29	50	20	54	40	14	305
	48	100	200	-195	60	59	58	47	-85	13	305
	49	-6	-106	289	34	35	36	-85	47	12	305
	490	-607	11	10	9	8	7	6	5	366	305
	305	305	305	305	305	305	305	305	305	305	305

The **magic sums** of above magic square are  $M_{4 \times 4} = 110$ ,  $S_{6 \times 6} = 150$ ,  $T_{8 \times 8} = 244$  and  $L_{10 \times 10} = 305$ . It is a magic square as it satisfies the condition given in (5), i.e., i.e.,  $T = \frac{4}{5} \times L$ . Moreover, the magic sums of magic rectangles are  $MR_{2 \times 4} = 40 \times 80$  and  $MR_{2 \times 4} = 94 \times 282$ .

**Result 5.14.** Let's consider following magic square of order 10 with *reduced entries*:

$10^*T-9^*L$	$27^*L-28^*T-A37-A36-A35-A33-A32-A31-A29-A30-A27-A28-A24-A25-A26-A23-A34$	A31	A32	A33	A34	A35	A36	A37	$18^*T-17^*L+A29+A30+A27+A28+A24+A25+A26+A23$
A23-T	$4^*S-6^*M+A11$	$S+2^*A12+2^*A13-4^*A11-M$	$2^*A11+9^*M-6^*S-A12-A13$	S-A12-M	S-A13-M	A11	T-S-A15	A15	L-A23
A24	$4^*M-3^*S+A8+A9+A10$	A1	$A4+M-A6-A1$	$A6-A3-A4$	A3	$4^*S-A8-A9-A10-5^*M$	$T-8^*S-A15-A1-A12-A8-A13+2^*A11-3^*A4+10^*M$	$A15+A1+A12+A8+A13-2^*A11+7^*S+3^*A4-10^*M$	L-T-A24
A25	S-A8-M	A6	A4	A5	$M-A4-A5-A6$	A8	$2^*A15+A17+A18+A1-2^*T+9^*S+A12+A8+A13-2^*A11+3^*A4-10^*M+A16$	$3^*T-10^*S-A17-A18-2^*A15-A1-A12-A8-A13+2^*A11-3^*A4+10^*M-A16$	L-T-A25
A26	S-A9-M	$A2+A3-A6$	$A1+A2-A5$	$M-A1-A2-A4$	$A4+A5+A6-A2-A3$	A9	T-S-A16	A16	L-T-A26
A27	S-A10-M	$M-A1-A2-A3$	$A5-A2-2^*A4+A6$	$A1+A2+A3+2^*A4-A5-A6$	A2	A10	T-S-A17	A17	L-T-A27
A28	$5^*M-A11-3^*S$	$4^*A11-2^*A12-2^*A13$	$7^*S+A12+A13-2^*A11-10^*M$	A12	A13	$5^*M-A11-3^*S$	T-S-A18	A18	L-T-A28
A29	$T-S+A12+A8+A13-2^*A11+2^*A1+7^*S+6^*A4-11^*M$	$8^*S+A12+A8+A13-2^*A11+2^*A1+6^*A4-11^*M$	$A20+A19+A21-T-2^*A12-2^*A8-2^*A13+4^*A11-4^*A1-14^*S-12^*A4+22^*M$	T-S-A19	T-S-A20	T-S-A21	(T-S)/2	$(7^*S-5^*T)/2$	L-T-A29
A30	$11^*M-A12-A8-A13+2^*A11-2^*A1-7^*S-6^*A4$	$T-9^*S-A12-A8-A13+2^*A11-2^*A1-6^*A4+11^*M$	$2^*T+13^*S+2^*A12+2^*A8+2^*A13-4^*A11+4^*A1+12^*A4-22^*M-A20-A19-A21$	A19	A20	A21	$(7^*S-5^*T)/2$	(T-S)/2	L-T-A30
$10^*L-9^*T-A23-A24-A25-A26-A27-A28-A29-A30$	$27^*T+A37+A36+A35+A34+A33+A32+A31+A29+A30+A27+A28+A24+A25+A26+A23-26^*L$	L-T-A31	L-T-A32	L-T-A33	L-T-A34	L-T-A35	L-T-A36	L-T-A37	$10^*L-11^*T$

*Details:*

It is a **single-digit bordered semi-magic** square of order 10 embedded with **cornered** magic square of order 8. The inner block is again a **single-digit bordered** magic square of order 6 with magic square of order 4 at the middle. In order to bring it as a magic square we need the same condition as given in (5).

Below are three examples based on the Result 5.14. First example is of **semi-magic** square and the second example as a magic square obtained by applying the condition given in (5).

**Example 5.23.** Let's consider a following **semi-magic** square based on the Result 5.14:

4a	semi	T=(4/5)*L									210
	-330	-60	101	104	107	110	113	116	119	-110	270
	-133	41	68	-9	6	3	41	7	53	193	270
	80	55	11	83	-11	17	-5	-155	215	-20	270
	83	18	26	20	23	31	32	325	-265	-23	270
	86	15	5	2	55	38	35	4	56	-26	270
	89	12	58	-5	33	14	38	1	59	-29	270
	92	9	-18	59	44	47	9	-2	62	-32	270
	95	193	283	-272	-5	-8	-11	30	0	-35	270
	98	-133	-223	332	65	68	71	0	30	-38	270
	110	120	-41	-44	-47	-50	-53	-56	-59	390	270
	270	270	270	270	270	270	270	270	270	270	270

The **magic** and **semi-magic** sums of above magic square are  $M_{4 \times 4} = 100$ ,  $S_{Sm_{6 \times 6}} = 150$ ,  $T_{8 \times 8} = 210$  and  $L_{Sm_{12 \times 12}} = 270$ . In this case there two magic rectangles with magic sums  $MR_{2 \times 6} = 60 \times 180$

**Example 5.24.** Let's consider a following magic square with **even** number magic sum based on the Result 5.14:

4b	mgc	T=(4/5)*L									330
	-330	48	101	104	107	110	113	116	119	-158	330
	-187	1	78	51	16	13	41	11	53	253	330
	80	65	11	123	-11	17	-5	-101	165	-14	330
	83	28	26	20	23	71	32	267	-203	-17	330
	86	25	5	2	95	38	35	8	56	-20	330
	89	22	98	-5	33	14	38	5	59	-23	330
	92	59	-18	9	44	47	59	2	62	-26	330
	95	107	243	-146	-1	-4	-7	32	40	-29	330
	98	-43	-179	210	65	68	71	40	32	-32	330
	224	18	-35	-38	-41	-44	-47	-50	-53	396	330
	330	330	330	330	330	330	330	330	330	330	330

The **magic sums** of above magic square are  $M_{4 \times 4} = 140$ ,  $S_{6 \times 6} = 200$ ,  $T_{8 \times 8} = 264$  and  $L_{10 \times 10} = 330$ . It is a magic square as it satisfies the conditions given in (5). Moreover, in this case there are two magic rectangles with magic sums  $MR_{2 \times 6} = 64 \times 192$ .

**Example 5.25.** Let's consider a following magic square with **odd** number magic sum based on the Result 5.14:

4c	mgc	T=(4/5)*L									295
	-295	-113	101	104	107	110	113	116	119	-67	295
	-159	1	78	51	16	13	41	-17	53	218	295
	80	65	11	123	-11	17	-5	-129	165	-21	295
	83	28	26	20	23	71	32	323	-287	-24	295
	86	25	5	2	95	38	35	-20	56	-27	295
	89	22	98	-5	33	14	38	-23	59	-30	295
	92	59	-18	9	44	47	59	-26	62	-33	295
	95	79	243	-118	-29	-32	-35	18	110	-36	295
	98	-43	-207	154	65	68	71	110	18	-39	295
	126	172	-42	-45	-48	-51	-54	-57	-60	354	295
	295	295	295	295	295	295	295	295	295	295	295



The **magic sums** of above magic square are  $M_{4 \times 4} = 140$ ,  $S_{6 \times 6} = 200$ ,  $T_{8 \times 8} = 236$  and  $L_{10 \times 10} = 295$ . It is a magic square as it satisfies the conditions given in (5). Moreover, in this case there are two magic rectangles with magic sums  $MR_{2 \times 6} = 36 \times 108$ .

**Result 5.15.** Let's consider following magic square of order 10 with *reduced entries*:

$10^*T-9^*L$	$27^*L-28^*T-A40-A39-A38-A36-A35-A34-A32-A33-A30-A31-A27-A28-A29-A26-A37$	A34	A35	A36	A37	A38	A39	A40	$A32+A33+A30+A31+A27+A28+A29+A26+18^*T-17^*L$
A26-T	T-S-A23	$2^*T+3^*S+A14-A20-A15$	T-S-A20	$A24+2^*A23-7^*T+2^*S+A22+A21+2^*A20+A16+A17+A19+2^*A15+A18$	T-S-A21	T-S-A22	T-S-A24	T-A16-A17-A19-A15-A18-A23-A14	L-A26
A27	T-S-A15	A1	$A4+M-A6-A1$	$A6-A3-A4$	A3	S-M-A10	A10	A15	L-T-A27
A28	T-S-A16	A6	A4	A5	$M-A4-A5-A6$	$A10+A11+A12-S+M$	$2^*S-2^*M-A10-A11-A12$	A16	L-T-A28
A29	T-S-A17	$A2+A3-A6$	$A1+A2-A5$	$M-A1-A2-A4$	$A4+A5+A6-A2-A3$	S-M-A11	A11	A17	L-T-A29
A30	T-S-A18	$M-A1-A2-A3$	$A5-A2-2^*A4+A6$	$A1+A2+A3+2^*A4-A5-A6$	A2	S-M-A12	A12	A18	L-T-A30
A31	T-S-A19	S-M-A8	$2^*S-A8-A1-3^*A4-2^*A10-A11-A12-M$	$2^*A8+A1+A9+3^*A4+2^*A10+A11+A12+M-2^*S$	S-M-A9	$(S-M)/2$	$(5^*M-3^*S)/2$	A19	L-T-A31
A32	T-S-A14	A8	$A8+A1+3^*A4+2^*A10+A11+A12-S$	$3^*S-2^*A8-A1-A9-3^*A4-2^*A10-A11-A12-2^*M$	A9	$(5^*M-3^*S)/2$	$(S-M)/2$	A14	L-T-A32
A33	$A23+A14+7^*S+A16+A17+A19+A15+A18-6^*T$	$A20+A15-T-A14-4^*S$	A20	$8^*T-A24-2^*A23-3^*S-A22-A21-2^*A20-A16-A17-A19-2^*A15-A18$	A21	A22	A24	A23	L-T-A33
$10^*L-9^*T-A26-A27-A28-A29-A30-A31-A32-A33$	$27^*T+A40+A39+A38+A36+A35+A34+A32+A33+A30+A31+A27+A28+A29+A26-26^*L+A37$	L-T-A34	L-T-A35	L-T-A36	L-T-A37	L-T-A38	L-T-A39	L-T-A40	$10^*L-11^*T$

*Details:*

It is a **single-digit bordered semi-magic** square of order 10 embedded with **cornered** magic square of order 6. These are **semi-magic** at one diagonal. In order to bring it as a magic square we need two conditions. These are as follows:

$$T = \frac{4}{5} \times L \quad \text{and} \quad S = \frac{3}{4} \times T, \quad (6)$$

where the letters M, S, T and L represents magic squares of orders 4, 6, 8 and 10.

Below are three examples based on the Result 5.15. First example is of **semi-magic** squares and the second example as a magic square obtained by applying the conditions given in (6).

**Example 5.26.** Let's consider a following **semi-magic** square based on the Result 5.15:

5a	semi	S=(3/4)*T and T=(4/5)*L									730
	-170	-1015	143	147	151	155	159	163	167	430	330
	-169	-59	1189	-47	-377	-51	-55	-63	-257	219	330
	115	-27	11	101	-11	19	73	47	67	-65	330
	119	-31	31	23	27	39	33	87	71	-69	330
	123	-35	3	-1	71	47	69	51	75	-73	330
	127	-39	75	-3	33	15	65	55	79	-77	330
	131	-43	81	41	41	77	60	-60	83	-81	330
	135	-23	39	79	79	43	-60	60	63	-85	330
	139	537	-1149	87	417	91	95	103	99	-89	330
	-220	1065	-93	-97	-101	-105	-109	-113	-117	220	330
	330	330	330	330	330	330	330	330	330	330	330

The **magic** and **semi-magic sums** of above magic square are  $M_{4 \times 4} = 120$ ,  $S_{Sm_{6 \times 6}} = 240$ ,  $T_{8 \times 8} = 280$  and  $L_{Sm_{10 \times 10}} = 330$ . In this case the magic rectangles sums is given as  $MR_{2 \times 4} = 120 \times 240$ .

**Example 5.27.** Let's consider a following magic square with **even** number magic sum based on the Result 5.15:

5b	mgc	S=(3/4)*T and T=(4/5)*L									330
	-330	-567	143	147	151	155	159	163	167	142	330
	-153	-33	1031	-21	-349	-25	-29	-37	-273	219	330
	115	-1	11	101	-11	19	31	47	67	-49	330
	119	-5	31	23	27	39	75	3	71	-53	330
	123	-9	3	-1	71	47	27	51	75	-57	330
	127	-13	75	-3	33	15	23	55	79	-61	330
	131	-17	39	-43	125	35	39	3	83	-65	330
	135	3	39	121	-47	43	3	39	63	-69	330
	139	339	-965	87	415	91	95	103	99	-73	330
	-76	633	-77	-81	-85	-89	-93	-97	-101	396	330
	330	330	330	330	330	330	330	330	330	330	330

The magic sums of above magic square are  $M_{4 \times 4} = 120$ ,  $S_{5m_{6 \times 6}} = 198$ ,  $T_{8 \times 8} = 264$  and  $L_{10 \times 10} = 330$ . It is a magic square as it satisfies the conditions given in (6). Moreover, in this case the magic rectangles sum is given as  $MR_{2 \times 4} = 78 \times 156$ .

**Example 5.28.** Let's consider a following magic square with **odd** number magic sum based on the Result 5.15:

5c	mgc	S=(3/4)*T and T=(4/5)*L									325
	-325	-590	143	147	151	155	159	163	167	155	325
	-149	-34	1014	-22	-327	-26	-30	-38	-277	214	325
	115	-2	11	102	-11	19	27	47	67	-50	325
	119	-6	31	23	27	40	79	-5	71	-54	325
	123	-10	3	-1	72	47	23	51	75	-58	325
	127	-14	76	-3	33	15	19	55	79	-62	325
	131	-18	35	-50	132	31	37	10	83	-66	325
	135	2	39	124	-58	43	10	37	63	-70	325
	139	342	-949	87	392	91	95	103	99	-74	325
	-90	655	-78	-82	-86	-90	-94	-98	-102	390	325
	325	325	325	325	325	325	325	325	325	325	325

The magic sums of above magic square are  $M_{4 \times 4} = 121$ ,  $S_{5m_6 \times 6} = 195$ ,  $T_{8 \times 8} = 260$  and  $L_{10 \times 10} = 325$ . It is a magic square as it satisfies the conditions given in (6). Moreover, in this case the magic rectangles sum is given as  $MR_{2 \times 4} = 74 \times 148$ .

**Result 5.16.** Let's consider following magic square of order 10 with *reduced entries*:

$10*T+A46-9*L$	$27*L-2*A46-28*T-A45-A44-A43-A41-A40-A39-A37-A38-A35-A36-A32-A33-A34-A31-A42$	A39	A40	A41	A42	A43	A44	A45	$A46+A38+A37+A36+A35+A34+A33+A32+A31+18*T-17*L$
A31-T	T-S-A27	$2*T+3*S+A17-2*A14-A24-2*A13-2*A8-5*M-3*A4-A18-2*A15$	T-S-A24	$A28+2*A27+2*A14-7*T+2*S+A26+A25+2*A24+2*A13+2*A8+5*M+3*A4+A19+A20+A22+2*A18+2*A15+A21$	T-S-A25	T-S-A26	T-S-A28	T-A19-A20-A22-A18-A21-A27-A17	L-A31
A32	T-S-A18	S-M-A15	S-A13-M	$2*A11-3*A4-A13-A8+A9+M$	$3*A4-3*S+2*A13+A14+2*A8+A9-A10+2*A15+2*M$	S-A14-M	S-A8-A9-A10-A11-A15	A18	L-T-A32
A33	T-S-A19	S-M-A9	A1	A4+M-A6-A1	A6-A3-A4	A3	A9	A19	L-T-A33
A34	T-S-A20	S-M-A10	A6	A4	A5	M-A4-A5-A6	A10	A20	L-T-A34
A35	T-S-A21	S-M-A11	A2+A3-A6	A1+A2-A5	M-A1-A2-A4	A4+A5+A6-A2-A3	A11	A21	L-T-A35
A36	T-S-A22	S-M-A8	M-A1-A2-A3	A5-A2-2*A4+A6	$A1+A2+A3+2*A4-A5-A6$	A2	A8	A22	L-T-A36
A37	T-S-A17	$A8-4*S+A9+A10+A11+A15+5*M$	A13	$S-2*A11+3*A4+A13+A8-A9-2*M$	$4*S+A11-3*A4-2*A13-A14-2*A8-A10-2*A15-3*M$	A14	A15	A17	L-T-A37
A38	$A27+A17+7*S+A19+A20+A22+A18+A21-6*T$	$A24+2*A13+2*A8+5*M+3*A4+A18+2*A15-T-A17+2*A14-4*S$	A24	$8*T-A28-2*A27-2*A14-3*S-A26-A25-2*A24-2*A13-2*A8-5*M-3*A4-A19-A20-A22-2*A18-2*A15-A21$	A25	A26	A28	A27	L-T-A38
$10*L-9*T-A31-A32-A33-A34-A35-A36-A37-A38-A46$	$27*T+2*A46+A45+A44+A43+A41+A40+A39+A37+A38+A35+A36+A32+A33+A34+A31-26*L+A42$	L-T-A39	L-T-A40	L-T-A41	L-T-A42	L-T-A43	L-T-A44	L-T-A45	$10*L-11*T-A46$

### Details:

It is a *single-digit bordered semi-magic* square of order 10 embedded with magic squares of orders 4, 6 and 8. These are *semi-magic* only at one diagonal. In order to bring it as a magic square we need three conditions. These are as follows:

$$T = \frac{4}{5} \times L \quad S = \frac{3}{4} \times T \quad \text{and} \quad M = \frac{3}{4} \times S, \quad (7)$$

where the letters  $M$ ,  $S$ ,  $T$  and  $L$  represents magic squares of orders 4, 6, 8 and 10.

Below are three examples based on the Result 5.16. First example is of **semi-magic** squares and the second and third examples are magic square obtained by applying the conditions given in (7), where the magic sum for the second example is **even** number and for the third example is **odd** number.

**Example 5.29.** Let's consider a following **semi-magic** square based on the Result 5.16:

6a	semi	M=(2/3)*S, S=(3/4)*T and T=(4/5)*L									300
	-79	-577	87	89	91	93	95	97	99	225	220
	-109	-13	-132	-7	538	-9	-11	-15	-171	149	220
	73	5	-9	-5	78	94	-7	-21	45	-33	220
	75	3	3	11	85	-11	15	27	47	-35	220
	77	1	1	21	17	19	43	29	49	-37	220
	79	-1	-1	7	5	59	29	31	51	-39	220
	81	-3	5	61	-7	33	13	25	53	-41	220
	83	7	131	35	-48	-64	37	39	43	-43	220
	85	181	182	57	-488	59	61	65	63	-45	220
	-145	617	-47	-49	-51	-53	-55	-57	-59	119	220
	220	220	220	220	220	220	220	220	220	220	220

The **magic** and **semi-magic** sums of above magic square are  $M_{4 \times 4} = 100$ ,  $S_{Sm_{6 \times 6}} = 130$ ,  $T_{Sm_{8 \times 8}} = 180$  and  $L_{Sm_{10 \times 10}} = 220$ .

**Example 5.30.** Let's consider a following magic square with **even** number magic sum based on the Result 5.16:



6b	mgc	M=(2/3)*S, S=(3/4)*T and T=(4/5)*L									360
	-259	179	87	89	91	93	95	97	99	-211	360
	-217	9	122	15	174	13	11	7	-63	289	360
	73	27	33	37	122	-76	35	65	45	-1	360
	75	25	45	11	129	-11	15	27	47	-3	360
	77	23	43	21	17	19	87	29	49	-5	360
	79	21	41	7	5	103	29	31	51	-7	360
	81	19	47	105	-7	33	13	25	53	-9	360
	83	29	7	35	-50	148	37	39	43	-11	360
	85	135	-50	57	-102	59	61	65	63	-13	360
	283	-107	-15	-17	-19	-21	-23	-25	-27	331	360
	360	360	360	360	360	360	360	360	360	360	360

The magic sums of above magic square are  $M_{4 \times 4} = 144$ ,  $S_{6 \times 6} = 216$ ,  $T_{8 \times 8} = 288$  and  $L_{10 \times 10} = 360$ . It is a magic square as it satisfies the conditions given in (7).

**Example 5.31.** Let's consider a following magic square with **odd** number magic sum based on the Result 5.16:

6c	mgc	M=(2/3)*S, S=(3/4)*T and T=(4/5)*L									345
	-244	110	87	89	91	93	95	97	99	-172	345
	-205	6	101	12	210	10	8	4	-75	274	345
	73	24	30	34	116	-61	32	56	45	-4	345
	75	22	42	11	123	-11	15	27	47	-6	345
	77	20	40	21	17	19	81	29	49	-8	345
	79	18	38	7	5	97	29	31	51	-10	345
	81	16	44	99	-7	33	13	25	53	-12	345
	83	26	13	35	-47	130	37	39	43	-14	345
	85	144	-32	57	-141	59	61	65	63	-16	345
	241	-41	-18	-20	-22	-24	-26	-28	-30	313	345
	345	345	345	345	345	345	345	345	345	345	345

The magic sums of above magic square are  $M_{4 \times 4} = 138$ ,  $S_{6 \times 6} = 207$ ,  $T_{8 \times 8} = 276$  and  $L_{10 \times 10} = 345$ . It is a magic square as it satisfies the conditions given in (7).

**Result 5.17.** *Let's consider following magic square of order 10 with reduced entries:*

10*T-9*L	27*L-28*T-A49-A48-A47-A45-A44-A43-A41-A42-A39-A40-A36-A37-A38-A35-A46	A43	A44	A45	A46	A47	A48	A49	A41+A42+A39+A40+A36+A37+A38+A35+18*T-17*L
A35-T	A19+A20+A21+A22+A23-A4-A7-A11-A15	A4	A7	A11	A15	S-A19-A20-A21-A22-A23	T-S-A29	A29	L-A35
A36	S-A12-A5-A8-A16-A19	A5	A8	A12	A16	A19	T-S-A30	A30	L-T-A36
A37	A4+A7+A11+A15-A20-A21-A22-A23-A1-A2-A3+A5+A8+A12+A16	S-A4-A5-A12+A21+A22+A23-A17-A7-A8-A15-A16-A13-A9-A11+A1+A2+A3	A9	A13	A17	A20	T-S-A31	A31	L-T-A37
A38	A1	S+A16+A13+A6-A19+A3-A1-A18-A14-2*A21-A22-A23-A20	A21+A22+A23+A20-A16-A13-A6+A19-A3	A14	A18	A21	T-S-A32	A32	L-T-A38
A39	A2	A6	S+A16+A13-A9+A6-A19+A3-A10-A21-A22-A23-A20-A7-A8	A5+A14+A10+2*A21+A22+3*A23+2*A20-A4+A8-A15-A16-A13+2*A9-2*A6+2*A19-A11-A2-A3-S	S-A19-A20-A21-A22-2*A23+A4+A7+A11+A15-A5-A9-A14	A22	T-S-A33	A33	L-T-A39
A40	A3	A12+A18+A14+A21+A20+A17+A7+A8+A15+A9-2*A6+A19+A11-A2-2*A3-S	A10	2*S+A4-A8+A15+A16-2*A9+2*A6-2*A19+A2+A3-A12-A5-2*A14-A10-2*A21-A22-3*A23-2*A20	A19+A20+A21+A22+2*A23-A4-A7-A11+A5+A9+A14-2*A15-A16-A17-A18	A23	2*S-2*T+A29+A30+A31+A32+A33	3*T-3*S-A29-A30-A31-A32-A33	L-T-A40
A41	T-S-A25	A30-A29-2*A9+A13-A25+2*S-2*A21-A22-3*A23-A18-2*A19-3*A20-A14+A15+A16-2*A10+A2+A4+A11-A5+2*A6-A8+A3	T-S-A26	T-S-A27	T-S-A28	2*A25+A28+A27+A26-A30+A29+2*A9-A13-T-S+2*A21+A22+3*A23+A18+2*A19+3*A20+A14-A15-A16+2*A10-A2-A4-A11+A5-2*A6+A8-A3	(T-S)/2	(7*S-5*T)/2	L-T-A41
A42	A25	A29-A30+2*A9-A13+T+A25-A11-3*S+2*A21+A22+3*A23+A18+2*A19+3*A20+A14-A15-A16+2*A10-A2-A4+A5-2*A6+A8-A3	A26	A27	A28	A30-A29-A28-A27-A26-2*A9+A13+2*T-2*A25-2*A21-A22-3*A23-A18-2*A19-3*A20-A14+A15+A16-2*A10+A2+A4+A11-A5+2*A6+A8-A3	(7*S-5*T)/2	(T-S)/2	L-T-A42
10*L-9*T-A35-A36-A37-A38-A39-A40-A41-A42	27*T+A49+A48+A47+A45+A44+A43+A41+A42+A39+A40+A36+A37+A38+A35-26*L+A46	L-T-A43	L-T-A44	L-T-A45	L-T-A46	L-T-A47	L-T-A48	L-T-A49	10*L-11*T

*Details:*

*It is a single-digit bordered semi-magic square of order 10 embedded with a cornered magic squares of orders 8 containing magic*

square of order 6 at the upper-left corner. These are **semi-magic** only at one diagonal. In order to bring it as a magic square we need a condition given in (5). The letters T and L represents magic squares of orders 8 and 10.

Below are two examples based on the Result 5.17. First example is of **semi-magic** squares and the second example is magic square obtained by applying the condition given in (5).

**Example 5.32.** Let's consider a following **semi-magic** square based on the Result 5.17:

7a	semi	10x10 <a href="https://numbers-magic.com/">https://numbers-magic.com/</a> ©IJT									310	7b	mgc	T=(4/5)*L									275
	-230	-955	143	147	151	155	159	163	167	370	270		-275	-820	143	147	151	155	159	163	167	285	275
	-109	78	14	17	21	25	-5	31	39	159	270		-109	78	14	17	21	25	-5	31	39	164	275
	115	40	15	18	22	26	29	30	40	-65	270		115	40	15	18	22	26	29	30	40	-60	275
	119	-4	55	19	23	27	30	29	41	-69	270		119	-4	55	19	23	27	30	29	41	-64	275
	123	11	-21	77	24	28	31	28	42	-73	270		123	11	-21	77	24	28	31	28	42	-68	275
	127	12	16	-1	110	-19	32	27	43	-77	270		127	12	16	-1	110	-19	32	27	43	-72	275
	131	13	71	20	-50	63	33	65	5	-81	270		131	13	71	20	-50	63	33	65	5	-76	275
	135	35	-72	34	33	32	148	35	-25	-85	270		135	35	-72	34	33	32	148	35	-25	-80	275
	139	35	142	36	37	38	-78	-25	35	-89	270		139	35	142	36	37	38	-78	-25	35	-84	275
	-280	1005	-93	-97	-101	-105	-109	-113	-117	280	270		-230	875	-88	-92	-96	-100	-104	-108	-112	330	275
	270	270	270	270	270	270	270	270	270	270	270		275	275	275	275	275	275	275	275	275	275	275

The **magic** and **semi-magic** sums of above magic square are

**First Example**

$$M_{6 \times 6} = 150$$

$$T_{8 \times 8} = 220$$

$$L_{10 \times 10} = 270$$

**Second Example**

$$M_{6 \times 6} = 150$$

$$T_{8 \times 8} = 220$$

$$L_{10 \times 10} = 275.$$

The magic rectangles sums are:

**First Example**

$$MR_{2 \times 6} = 70 \times 210$$

**Second Example**

$$MR_{2 \times 6} = 70 \times 210.$$

**Result 5.18.** *Let's consider following magic square of order 10 with reduced entries:*

10*T-9*L	27*L-28*T-A51-A50-A49-A47-A46-A45-A43-A44-A41-A42-A38-A39-A40-A37-A48	A45	A46	A47	A48	A49	A50	A51	A43+A44+A41+A42+A38+A39+A40+A37+18*T-17*L
A37-T	T-S-A28	2*T+3*S+A30-A25-A31	T-S-A25	A29+2*A28-7*T+2*S+A27+A26+2*A25+A32+A33+A35+2*A31+A34	T-S-A26	T-S-A27	T-S-A29	T-A32-A33-A35-A31-A34-A28-A30	L-A37
A38	T-S-A31	A19+A20+A21+A22+A23-A4-A7-A11-A15	A4	A7	A11	A15	S-A19-A20-A21-A22-A23	A31	L-T-A38
A39	T-S-A32	S-A12-A5-A8-A16-A19	A5	A8	A12	A16	A19	A32	L-T-A39
A40	T-S-A33	A4+A7+A11+A15-A20-A21-A22-A23-A1-A2-A3+A5+A8+A12+A16	S-A4-A5-A12+A21+A22+A23-A17-A7-A8-A15-A16-A13-A9-A11+A1+A2+A3	A9	A13	A17	A20	A33	L-T-A40
A41	T-S-A34	A1	S+A16+A13+A6-A19+A3-A1-A18-A14-2*A21-A22-A23-A20	A21+A22+A23+A20-A16-A13-A6+A19-A3	A14	A18	A21	A34	L-T-A41
A42	T-S-A35	A2	A6	S+A16+A13-A9+A6-A19+A3-A10-A21-A22-A23-A20-A7-A8	A5+A14+A10+2*A21+A22+3*A23+2*A20-A4+A8-A15-A16-A13+2*A9-2*A6+2*A19-A11-A2-A3-S	S-A19-A20-A21-A22-2*A23+A4+A7+A11+A15-A5-A9-A14	A22	A35	L-T-A42
A43	T-S-A30	A3	A12+A18+A14+A21+A20+A17+A7+A8+A15+A9-2*A6+A19+A11-A2-2*A3-S	A10	2*S+A4-A8+A15+A16-2*A9+2*A6-2*A19+A2+A3-A12-A5-2*A14-A10-2*A21-A22-3*A23-2*A20	A19+A20+A21+A22+2*A23-A4-A7-A11+A5+A9+A14-2*A15-A16-A17-A18	A23	A30	L-T-A43
A44	A28+A30+7*S+A32+A33+A35+A31+A34-6*T	A25+A31-T-A30-4*S	A25	8*T-A29-2*A28-3*S-A27-A26-2*A25-A32-A33-A35-2*A31-A34	A26	A27	A29	A28	L-T-A44
10*L-9*T-A37-A38-A39-A40-A41-A42-A43-A44	27*T+A51+A50+A49+A47+A46+A45+A43+A44+A41+A42+A38+A39+A40+A37-26*L+A48	L-T-A45	L-T-A46	L-T-A47	L-T-A48	L-T-A49	L-T-A50	L-T-A51	10*L-11*T

**Details:**

*It is a **single-digit bordered semi-magic** square of order 10 embedded with a magic squares of orders 6. It is **semi-magic** only at one diagonal. In order to bring it as a magic square we need two conditions given in (6). The letters S, T and L represents magic squares of orders 6, 8 and 10.*

Below are two examples based on the Result 5.18. First example is of **semi-magic** squares and the second example is a magic square obtained by applying the conditions given in (6).

**Example 5.33.** Let's consider a following **semi-magic** square based on the Result 5.18:

8a	semi	10x10 <a href="https://numbers-magic.com/">https://numbers-magic.com/</a> ©IJT									110	8b	mgc	T=(4/5)*L and S=(3/4)*T										250
	-430	880	55	56	57	58	59	60	61	-586	270		-250	340	55	56	57	58	59	60	61	-246	250	
	-153	12	814	15	-586	14	13	11	-93	223	270		-153	12	814	15	-586	14	13	11	-93	203	250	
	48	9	78	14	17	21	25	-5	41	22	270		48	9	78	14	17	21	25	-5	41	2	250	
	49	8	40	15	18	22	26	29	42	21	270		49	8	40	15	18	22	26	29	42	1	250	
	50	7	-4	55	19	23	27	30	43	20	270		50	7	-4	55	19	23	27	30	43	0	250	
	51	6	11	-21	77	24	28	31	44	19	270		51	6	11	-21	77	24	28	31	44	-1	250	
	52	5	12	16	-1	110	-19	32	45	18	270		52	5	12	16	-1	110	-19	32	45	-2	250	
	53	10	13	71	20	-50	63	33	40	17	270		53	10	13	71	20	-50	63	33	40	-3	250	
	54	143	-764	35	636	36	37	39	38	16	270		54	143	-764	35	636	36	37	39	38	-4	250	
	496	-810	15	14	13	12	11	10	9	500	270		296	-290	-5	-6	-7	-8	-9	-10	-11	300	250	
	270	270	270	270	270	270	270	270	270	270	270		250	250	250	250	250	250	250	250	250	250	250	

The **magic** and **semi-magic** sums of above magic square are

**First Example**

$$M_{6 \times 6} = 150$$

$$T_{8 \times 8} = 200$$

$$L_{10 \times 10} = 270$$

**Second Example**

$$M_{6 \times 6} = 150$$

$$T_{8 \times 8} = 200$$

$$L_{10 \times 10} = 250.$$

**Result 5.19.** Let's consider following magic square of order 10 with **reduced entries**:



10*T-9*L	27*L-28*T-A28-A27- A26-A24-A23-A22- A20-A21-A18-A19-A15- A16-A17-A14-A25	A22	A23	A24	A25	A26	A27	A28	A20+A21+A18+A19 +A15+A16+A17+ A14+18*T-17*L
A14-T	A1-A3-A5-A6-A8+A7	A6+A8- 2*m+A11+A1- A3-A5	2*m	2*m+2*A3+2*A5- A7-A11-2*A1	A5+2*A3-A7- A11+A10-A9	2*m-A9-A5- A10	A9- 2*A3+A7+A11	A9	L-A14
A15	m+A3+A5+A6+A8-A7- A1	3*m+A3+A5- A6-A8-A11-A1	(-m)	A7-m+A11+2*A1- 2*A3-2*A5	m+A9-A10+A11- A5-2*A3+A7	A9+A5+A10-m	m-A9+2*A3- A7-A11	m-A9	L-T-A15
A16	2*A3+2*A5-A7-A11-A1	A1	A6+A8+A11-A3- A5	2*m+A7-A3-A5- A6-A8	A5+2*A3-A7- A11	A5	2*m+A7+A11- A10-2*A3-2*A5	A10	L-T-A16
A17	A1+m-2*A3- 2*A5+A7+A11	m-A1	m+A3+A5-A6- A8-A11	A3+A5+A6+A8- A7-m	m+A7+A11-A5- 2*A3	m-A5	2*A3+2*A5-A7- A11+A10-m	m-A10	L-T-A17
A18	A3+A2-A7-A11+A4	2*m-A3-A2-A4	A7+A11-A3	A3	2*m-A6-A8-A11	A6+A8-A7	A7	A11	L-T-A18
A19	m+A7+A11-A4-A3-A2	A3+A2+A4-m	m+A3-A7-A11	m-A3	A6+A8+A11-m	m+A7-A6-A8	m-A7	m-A11	L-T-A19
A20	A2+2*A3-A7-A11	A2	2*m+A7+A11- 2*A2-A4-2*A3	A4	A6+A11-A7	A6	A8	2*m+A7-2*A6- A8-A11	L-T-A20
A21	m+A7+A11-A2-2*A3	m-A2	2*A3-A7- A11+2*A2+A4- m	m-A4	m+A7-A11-A6	m-A6	m-A8	2*A6+A8+A11- A7-m	L-T-A21
10*L-9*T-A14- A15-A16-A17- A18-A19-A20- A21	27*T+A28+A27+A26+ A24+A23+A22+A20+ A21+A18+A19+A15+A 16+A17+A14- 26*L+A25	L-T-A22	L-T-A23	L-T-A24	L-T-A25	L-T-A26	L-T-A27	L-T-A28	10*L-11*T

### Details:

It is a **single-digit bordered semi-magic** square of order 10 embedded with a **striped** magic square of order 8. It is **semi-magic** only at one diagonal. In order to bring it as a magic square we need a condition given in (5). The letters T and L represents magic squares of orders 8 and 10.

Below are two examples based on the Result 5.19. First example is of **semi-magic** squares and the second example is a magic square obtained by applying the conditions given in (5).

**Example 5.34.** Let's consider a following *semi-magic* square based on the Result 5.19:

9a	semi	10x10 <a href="https://numbers-magic.com/">https://numbers-magic.com/</a> ©IJT									60	9b	mgc	T=(4/5)*L									200
	-380	995	32	33	34	35	36	37	38	-640	220		-200	455	32	33	34	35	36	37	38	-300	200
	-136	-34	-42	80	76	4	26	31	19	196	220		-136	-34	-42	80	76	4	26	31	19	176	200
	25	74	82	-40	-36	36	14	9	21	35	220		25	74	82	-40	-36	36	14	9	21	15	200
	26	7	11	27	35	3	15	42	20	34	220		26	7	11	27	35	3	15	42	20	14	200
	27	33	29	13	5	37	25	-2	20	33	220		27	33	29	13	5	37	25	-2	20	13	200
	28	1	41	25	13	25	17	17	21	32	220		28	1	41	25	13	25	17	17	21	12	200
	29	39	-1	15	27	15	23	23	19	31	220		29	39	-1	15	27	15	23	23	19	11	200
	30	0	12	54	14	20	16	18	26	30	220		30	0	12	54	14	20	16	18	26	10	200
	31	40	28	-14	26	20	24	22	14	29	220		31	40	28	-14	26	20	24	22	14	9	200
	540	-935	28	27	26	25	24	23	22	440	220		340	-415	8	7	6	5	4	3	2	240	200
	220	220	220	220	220	220	220	220	220	220	220		200	200	200	200	200	200	200	200	200	200	200

The **magic** and **semi-magic** sums of above magic square are

**First Example**

$$T_{8 \times 8} = 160$$

$$L_{10 \times 10} = 220$$

**Second Example**

$$T_{8 \times 8} = 160$$

$$L_{10 \times 10} = 200.$$

In both the examples the magic sums of 8 magic rectangles appearing magic square of order 8 are of equal sums, i.e.,  $40 \times 80$ .

**Result 5.20.** Let's consider following magic square of order 10 with *reduced entries*:

$10*2*S+A35-9*L$	$27*L-2*A35-28*2*S-A34-A33-A32-A30-A29-A28-A26-A27-A24-A25-A21-A22-A23-A20-A31$	A28	A29	A30	A31	A32	A33	A34	$A35+A27+A26+A25+A24+A23+A22+A21+A20+18*2*S-17*L$
A20-2*S	$2*S-2*A4-2*A7-A12+A3+A6$	$3*A4+3*A7-A15-A2+A12-A3-A6-2*S$	$A2+A14-A16-4*A4-4*A7+A3+A6-A1+4*S+A15$	$A16+3*A4-3*S+3*A7-A3-A6+A1-A14$	A3	$2*S-A9-A4-A7-A5-A3$	$A5+A8-A16+A3+A6-A1+A9$	$A16+A4+A7-A3-A6+A1-A8-S$	L-A20
A21	A2	$A4+A7-A15-S$	A1	$2*S-A4-A7+A15-A1-A2$	A5	$S-A9-A4-A7$	$A16+2*A4-2*S+2*A7+A1-A10$	$A9-A4-A7+2*S-A16-A1+A10-A5$	L-2*S-A21
A22	$2*A4+2*A7-A13-S-A14+A16-A3-A6+A1-A2$	$4*S-3*A4-3*A7-A12+A3+A6-A13-A1$	$2*A4+2*A7+A1-2-A3-A6+A13+A15-2*S$	$A2+A13+A14-A16-A4-A7+A3+A6-A15$	$2*A4-S-A8+A16+A7-A3-A6+A1-A5$	$A3-A4+2*S-A16-2*A7-A1+A10$	$A9+A7-A3$	$A8+A3+A6-A10+A5-A9-A4$	L-2*S-A22
A23	$A12+A13+A14-A16-A1$	$A1-A4-A7+A13+2*A15+A2$	$2*A4+2*A7-A12-A13-A14-S+A16-2*A15-A2$	$2*S-A4-A7-A13$	$2*S-2*A4+A8-A16-A7+A6-A1$	$A16+3*A4-4*S+4*A7+A1+2*A9+A5-A10$	$3*S-A8-3*A7-A6-2*A9+A10-A5-2*A4$	A4	L-2*S-A23
A24	A6	$A9+S-A11-A6$	$A11-A8-A9$	A8	A12	$A15+S-A17-A12$	$A17-A14-A15$	A14	L-2*S-A24
A25	A11	A9	A10	$S-A9-A10-A11$	A17	A15	A16	$S-A15-A16-A17$	L-2*S-A25
A26	$A7+A8-A11$	$A6+A7-A10$	$S-A6-A7-A9$	$A9+A10+A11-A7-A8$	$A13+A14-A17$	$A12+A13-A16$	$S-A12-A13-A15$	$A15+A16+A17-A13-A14$	L-2*S-A26
A27	$S-A6-A7-A8$	$A10-A7-2*A9+A11$	$A6+A7+A8+2*A9-A10-A11$	A7	$S-A12-A13-A14$	$A16-A13-2*A15+A17$	$A12+A13+A14+2*A15-A16-A17$	A13	L-2*S-A27
$10*L-9*2*S-A20-A21-A22-A23-A24-A25-A26-A27-A35$	$27*2*S+2*A35+A34+A33+A32+A30+A29+A28+A26+A27+A24+A25+A21+A22+A23+A20-26*L+A31$	L-2*S-A28	L-2*S-A29	L-2*S-A30	L-2*S-A31	L-2*S-A32	L-2*S-A33	L-2*S-A34	$10*L-11*2*S-A35$

### Details:

It is a **single-digit bordered semi-magic** square of order 10 embedded with a **pandiagonal** magic square of order 8 divided in four equal sums magic squares of order 4. It is **semi-magic** only at one diagonal. In order to bring it as a magic square we need a condition given in (5). The letters T and L represents magic squares of orders 8 and 10.

Below are two examples based on the Result 5.20. First example is of **semi-magic** squares and the second example is a magic square obtained by applying the conditions given in (5).

**Example 5.35.** Let's consider a following *semi-magic* square based on the Result 5.20:

10a	semi	10x10 <a href="https://numbers-magic.com/">https://numbers-magic.com/</a> ©IJT										<del>230</del>	10b	mgc	T=(4/5)*L										<del>300</del>
		-345	1005	38	39	40	41	42	43	44	-637	310			-255	735	38	39	40	41	42	43	44	-467	300
		-210	185	-191	409	-283	13	162	44	-99	280	310			-210	185	-191	409	-283	13	162	44	-99	270	300
		31	12	-114	11	211	15	70	-161	196	39	310			31	12	-114	11	211	15	70	-161	196	29	300
		32	-109	360	-137	6	-100	188	23	9	38	310			32	-109	360	-137	6	-100	188	23	9	28	300
		33	32	65	-163	186	192	-300	214	14	37	310			33	32	65	-163	186	192	-300	214	14	27	300
		34	16	102	-16	18	22	96	-22	24	36	310			34	16	102	-16	18	22	96	-22	24	26	300
		35	21	19	20	60	27	25	26	42	35	310			35	21	19	20	60	27	25	26	42	25	300
		36	14	13	68	25	20	19	50	31	34	310			36	14	13	68	25	20	19	50	31	24	300
		37	69	-14	48	17	51	-20	66	23	33	310			37	69	-14	48	17	51	-20	66	23	23	300
		627	-935	32	31	30	29	28	27	26	415	310			527	-675	22	21	20	19	18	17	16	315	300
		310	310	310	310	310	310	310	310	310	310	<del>310</del>			300	300	300	300	300	300	300	300	300	300	<del>300</del>

The **magic** and **semi-magic** sums of above magic square are

**First Example**

$$T_{4 \times 4} = 120$$

$$T_{8 \times 8} = 240$$

$$L_{10 \times 10} = 310$$

**Second Example**

$$T_{4 \times 4} = 120$$

$$T_{8 \times 8} = 240$$

$$L_{10 \times 10} = 300.$$

## 6 Author's Contribution to Magic Squares and Recreation Numbers

For author's contribution to **magic squares** and **recreation numbers** lease see the links below:

- **Inder J. Taneja**, Magic Squares,
  - (i) <https://numbers-magic.com/?p=668>
  - (ii) <https://inderjtaneja.wordpress.com/2019/06/27/publications-magic-squares/>
- **Inder J. Taneja**, Recreation of Numbers,
  - (i) <https://numbers-magic.com/?p=671>
  - (ii) <https://inderjtaneja.wordpress.com/2019/06/27/publications-recreation-of-numbers/>

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- [14] **Inder J. Taneja**, Symmetric Properties of Nested Magic Squares, **Zenodo**, June 29, 2019, pp. 1-55, <http://doi.org/10.5281/zenodo.3262170>.
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